## Experiment 1: Determination of heat capacity coefficients $\boldsymbol{c}_{\boldsymbol{p}}$ and $\boldsymbol{c}_{\boldsymbol{v}}$ for air

## 1. Introduction

According to the definition, the heat capacity coefficients, $c_{v}$ and $c_{p}$ are given by the formulas:

$$
\begin{align*}
C_{v} & =\frac{Q_{(v) 1-2}}{m\left(T_{2}-T_{1}\right)}  \tag{1}\\
C_{p} & =\frac{Q_{(p) 1-2}}{m\left(T_{2}-T_{1}\right)} \tag{2}
\end{align*}
$$

Where:
$Q_{(v) 1-2}, Q_{(p) 1-2}$ are heat delivered to the system at constant volume and constant pressure respectively.
$m$ - mass of the thermodynamic medium
$T_{1}, T_{2}$ temperatures of the system at the beginning of the process (1) and at the end of the process (2)

The determination of $c_{v}$ and $c_{p}$ therefore, requires the measurement of all quantities occurring in the equations. This is a calorimetric method which gives accurate results for liquids and solids. For gases however, both in the low and high pressure, this method leads to large errors. Therefore, the dynamic method is used to determine $c_{v}$ and $c_{p}$ of gas with low pressure, using the adiabatic expansion process.

The adiabatic equation is:

$$
\begin{equation*}
p v^{k}=i d e m \tag{3}
\end{equation*}
$$

Where:
$k$ is an adiabatic constant, given by:

$$
\begin{equation*}
k=\frac{c_{p}}{c_{v}} \tag{3.1}
\end{equation*}
$$

The heat capacity coefficient can be expressed in relation the individual gas constant with the equation:

$$
\begin{equation*}
c_{p}-c_{v}=R \tag{4}
\end{equation*}
$$

Where:

R is an individual gas constant.

Combining equations 3.1 with 4 gives:

$$
\begin{align*}
c_{v} & =\frac{R}{k-1}  \tag{5}\\
c_{p} & =R \frac{k}{k-1} \tag{6}
\end{align*}
$$

## 2. Description of the experiment

The tank with constant volume is inflated with air from ambient pressure ( $p_{0}$ ) up to the absolute pressure ( $p_{1}=p_{0}+\Delta p_{1}$ ). The initial air temperature is $t_{1}=t_{0}$. Rapid opening and closing of a valve (for less than 1 second) connects the tank to the environment. In consequence, a rapid outflow of air into the environment occurs. The pressure in the tank drops to the value $p_{2}=$ $p_{0}$ and the temperature reaches the value $t_{2}<t_{0}$ (see fig 1). After closing the valve, wait until the air temperature $t_{3}$ equals with ambient temthe perature $t_{0}$, i.e. $t_{3}=t_{0}$ (and $\Delta p_{3}$ will stabilize). After this the final pressure should be noted.


Fig. 1. Schematic description of the process

During gas outflow from the tank, for obvious reasons, the loss of heat is unavoidable. However, it can be shown that in the considered range of temperature and pressure the indicator Y, which corresponds to the deviation from the adiabatic process, is relatively small.

$$
\begin{equation*}
Y=\frac{Q_{z 1-2}}{\left|U_{2}-U_{1}\right|} \tag{7}
\end{equation*}
$$

Where:
$Q_{z 1-2}$ is a total amount of heat added or removed to the system during the period of time $\Delta \tau$.
$\left|U_{2}-U_{1}\right|$ is a total change of the internal energy of the gas

The adiabatic process occurs when $Y=0$. In any other case, depending on the specific value of this indicator, it is said that the adiabatic process was realised with an accuracy given by $Y$

When $Q_{z} \neq 0$, a polytropic (not adiabatic) process is being carried out with an exponent $w \neq k$. Knowing, the value of the exponent " $w$ " the amount of heat $Q_{z}$ can be evaluated. The polytropic exponent ( $w$ ) can be calculated from the formula (8), in to which only the measured values appear.

$$
\begin{equation*}
w_{i}=\frac{\ln \left(p_{1 i} / p_{0}\right)}{\ln \left(p_{1 i} / p_{3 i}\right)} \tag{8}
\end{equation*}
$$

The experiment should be performed for several initial pressures $\left(p_{1 i}\right)$. In consequence, several final pressures shall be obtained ( $p_{3 i}$ )

$$
\begin{align*}
& p_{1 i}=p_{0}+\Delta p_{1 i}  \tag{9}\\
& p_{3 i}=p_{0}+\Delta p_{3 i} \tag{10}
\end{align*}
$$

Where:
$i$ is the subsequent measurement;
$\Delta p$ the measured manometric pressure

## 3. Results elaboration

For each " $i$ " measurement, calculate the " $w_{i}$ " value. Once the value of polytropic exponent is found for all measurements calculate its arithmetic average value- equation (11).

$$
\begin{equation*}
w_{a v}=\frac{\sum_{i} w_{i}}{i} \tag{11}
\end{equation*}
$$

Calculate the heat capacities $c_{p m}$ and $c_{v m}$ which corresponds to the polytropic (not adiabatic) process.

$$
\begin{align*}
c_{v m} & =\frac{R}{w_{a v}-1}  \tag{12}\\
c_{p m} & =R \frac{w_{a v}}{w_{a v}-1} \tag{13}
\end{align*}
$$

These values should be compared with the values calculated from formulas (5) and (6) for $k=$ 1.4.

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| No. | measurement \i | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{p}_{\mathrm{o}}, \mathrm{Pa} / \mathrm{p}_{\mathrm{o}}, \mathrm{mm} \mathrm{H}_{2} \mathrm{O}$ |  |  |  |  |  |
| 2 | $\Delta \mathrm{p}_{\mathrm{li}}, \mathrm{mm} \mathrm{H}_{2} \mathrm{O}$ | 800 | 650 | 500 | 350 | 200 |
| 3 | $p_{1 i}$ |  |  |  |  |  |
| 4 | $\Delta \mathrm{p}_{3 \mathrm{i}}, \mathrm{mm} \mathrm{H}_{2} \mathrm{O}$ |  |  |  |  |  |
| 5 | $p_{3 i}$ |  |  |  |  |  |
| 6 | $w_{i}$ |  |  |  |  |  |
| 7 | $w_{a v}$ |  |  |  |  |  |
| 8 | $c_{v m}$ |  |  |  |  |  |
| 9 | $c_{p m}$ |  |  |  |  |  |
| 10 | $\mathrm{c}_{\mathrm{v}}$ |  |  |  |  |  |
| 11 | $\mathrm{c}_{\mathrm{p}}$ |  |  |  |  |  |
| 12 | $\mathrm{c}_{\mathrm{vm}} / \mathrm{c}_{\mathrm{v}}$ |  |  |  |  |  |
| 13 | $\mathrm{c}_{\mathrm{pm}} / \mathrm{c}_{\mathrm{p}}$ |  |  |  |  |  |

measurement system 2 (big valve)

| No. | measurement \i | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{p}_{\mathrm{o}}, \mathrm{Pa} / \mathrm{p}_{\mathrm{o}}, \mathrm{mm} \mathrm{H}_{2} \mathrm{O}$ |  |  |  |  |  |
| 2 | $\Delta \mathrm{p}_{\mathrm{li}}, \mathrm{mm} \mathrm{H}_{2} \mathrm{O}$ | 800 | 650 | 500 | 350 | 200 |
| 3 | $p_{1 i}$ |  |  |  |  |  |
| 4 | $\Delta \mathrm{p}_{3} \mathrm{i}, \mathrm{mm} \mathrm{H} \mathrm{H}_{2} \mathrm{O}$ |  |  |  |  |  |
| 5 | $p_{3 i}$ |  |  |  |  |  |
| 6 | $w_{i}$ |  |  |  |  |  |
| 7 | $w_{a v}$ |  |  |  |  |  |
| 8 | $c_{v m}$ |  |  |  |  |  |
| 9 | $c_{p m}$ |  |  |  |  |  |
| 10 | $\mathrm{c}_{\mathrm{v}}$ |  |  |  |  |  |
| 11 | $\mathrm{c}_{\mathrm{p}}$ |  |  |  |  |  |
| 12 | $\mathrm{c}_{\mathrm{vm}} / \mathrm{c}_{\mathrm{v}}$ |  |  |  |  |  |
| 13 | $\mathrm{c}_{\mathrm{pm}} / \mathrm{c}_{\mathrm{p}}$ |  |  |  |  |  |

