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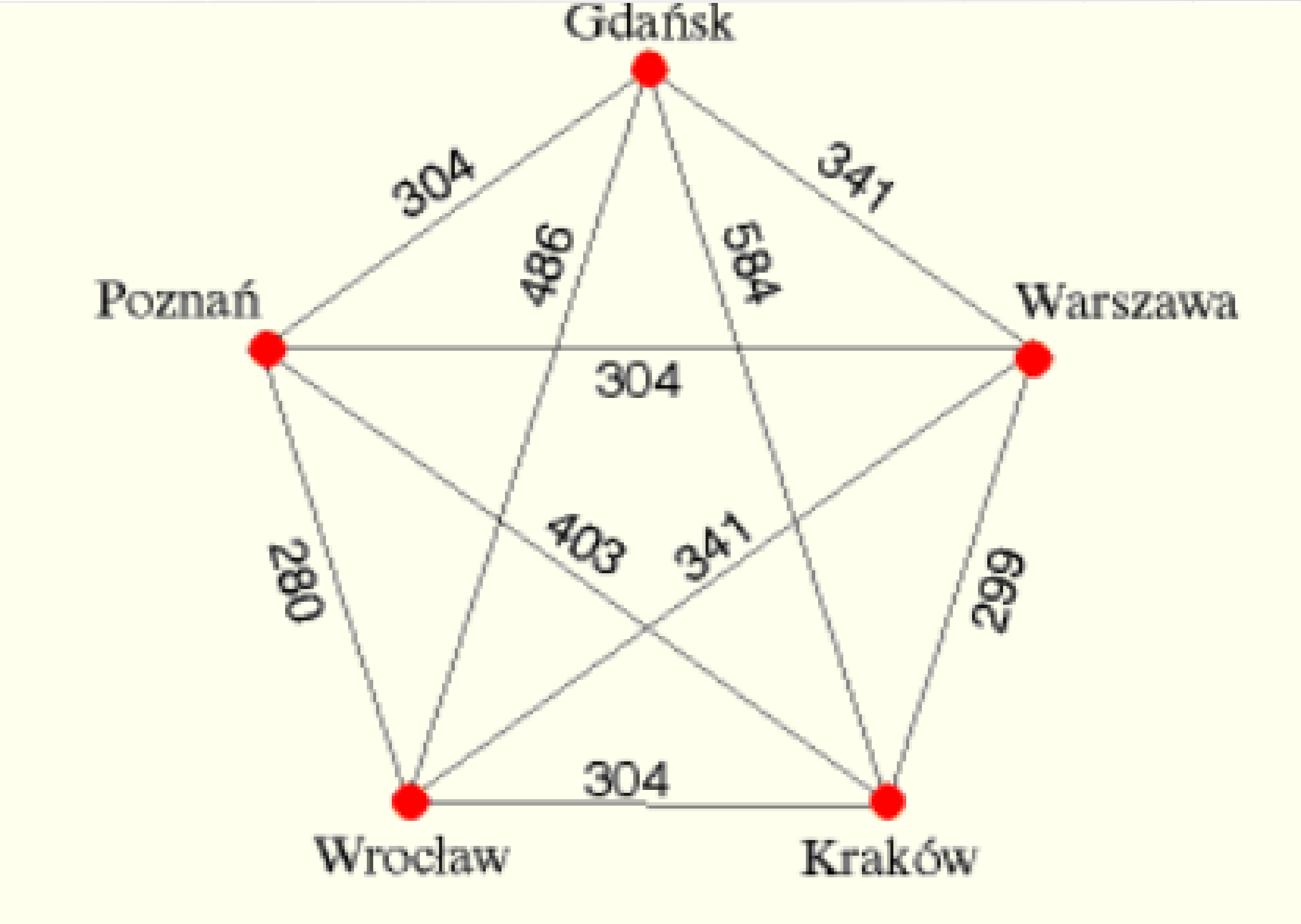
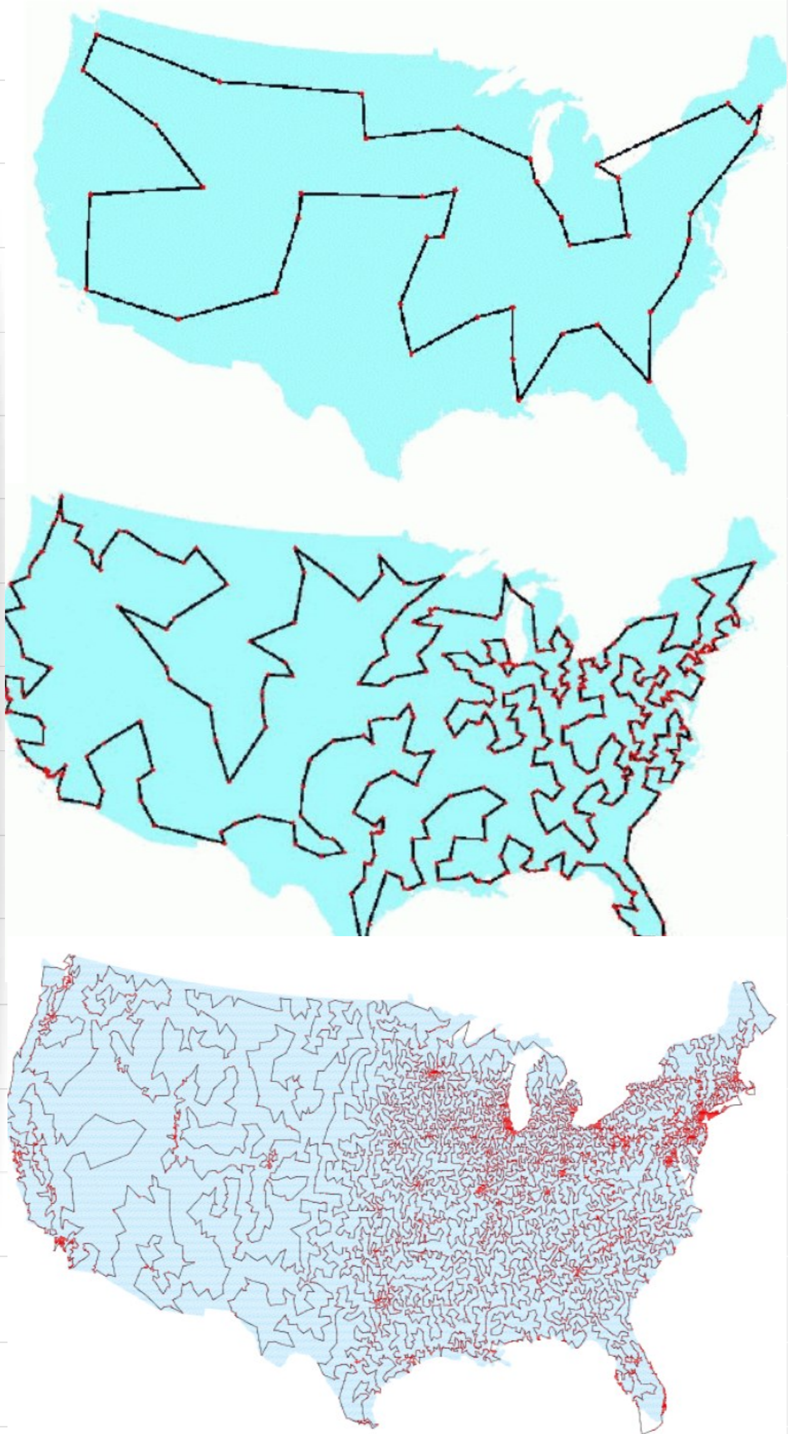
Basics of transportation systems

Travelling salesman problem, TSP

- The problem is impossible to solve
- Solving full Hamiltonian cycle and selection of the optimum solution is not reachable in realistic time
- Required amount of options to analyze is $\frac{n!}{2}$



For n=10 the amount of cycles is $10!/2 = 181440$
For n=20 the amount of cycles is $20!/2 = 10^{18}$



49 cities George Dantzig, Ray Fulkerson i Selmer Johnson (1954)

532 cities Padberg i Rinaldi (1987)

13549 cities solved in 1998 year

A transportation system can be defined as a set of elements and the interactions between them that produce both the **demand for travel** within a given area and the **provision of transportation services** to satisfy this demand.

Activities → **Transportation system** → **Demand**

Transportation system ↓ **Transportation Supply**

Activities

set of individual, social, and economic behaviors and interactions that give rise to travel **demand**:

- Goods transportation
- Touristic

Transportation system

Transportation Supply

component is made up of the facilities (roads, parking spaces, railway lines, etc.), services (transit lines and timetables), regulations (road circulation and parking regulations), and prices (transit fares, parking prices, road tolls, etc.) that produce travel opportunities.

Demand

the need to access urban functions and services in different places and use the transportation network so that is possible to undertake different activities (work, study, shopping, etc.) in different locations

- mobility choices
- travel choices (short /long)

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Transportation system

Transport is an important factor for sustainable social, economic, environmental, spatial and functional development.

It complies with:

- **Service** - the efficiency of the communication system depends on the primary scope of the public purposes,
- **Stimulating the development of the area** - by providing land and developing the supply of transport services in advance of the actual needs,
- **Inhibiting the development of the area** - where required - through the creation of transport barriers,
- **Spatial composition** - elements of the transport system, due to the vast extent of their influence, put on a par with the spatial solutions architectural and urban planning.

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Transportation system

- **The complexity** - the transport system forms a very large number of elements, and the reaction and interaction between these elements, and environment systm;

- **Probabilism** - can not be predicted of all phenomena, states, relationships and interaction occurring during the transport processes;

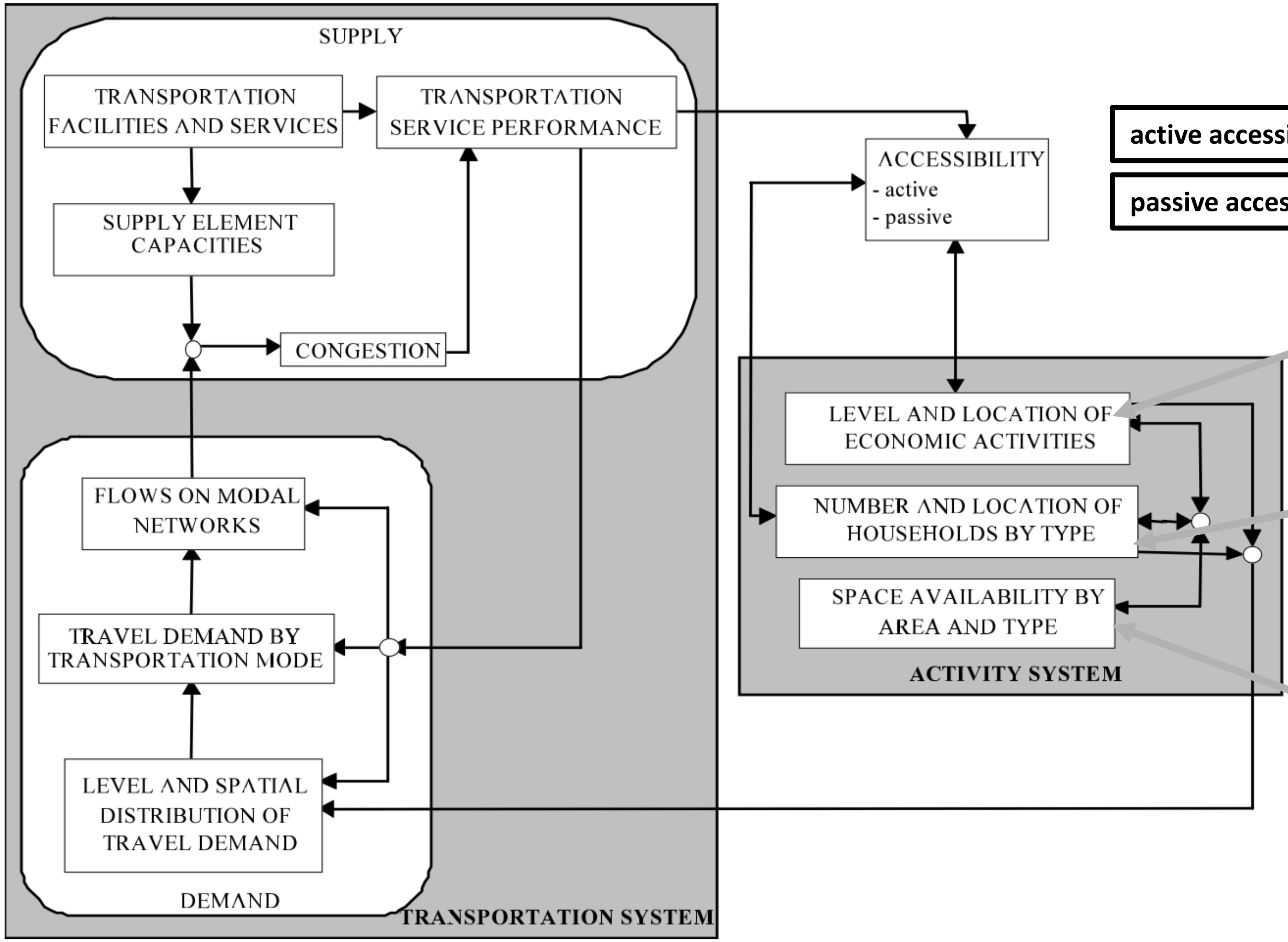
- **Limited ability to self-regulation** - transport system is created by man. It does not have the ability to self adjustment, hence it can be unreliable. Transportation system can not decide which solution is reliable and rejects the deviations

- **Dynamism** - results from human intervention in its structure and functioning and subject to change in time and space and adapts to the new conditions

The general approach of transportation systems engineering is to isolate the elements most relevant to a problem at hand, and to group these elements and the relationships be tween them within the analysis system. The transportation system of a given area can also be seen as a subsystem of a wider territorial system with which it strongly interacts.

Travel from one location to another frequently involves the successive use of several connected facilities or services. **Transportation facilities generally have a finite capacity**, that is, a maximum numer of units that may use them in a given time interval.

the trips made by people between the different zones of the city, for different purposes, in different periods of the day, by means of the different available transportation modes. Similarly, economic activities require the transportation of goods that are consumed by other activities or by households. Goods are moved between production plants, retail locations, and houses or other "final consumption" sites. Their movementsmake up freight travel demand and corresponding flows



active accessibility- reaching other zones

passive accessibility being reached from other zones

socio-economic indicators (e.g., sector of activity; value added; number of employees)

categorized by factors such as income level, life-cycle, composition,

The real estate system, characterized by the floor space available in each zone for various uses (industrial production, offices, building areas, etc.) and the associated market prices

Transportation system engineering

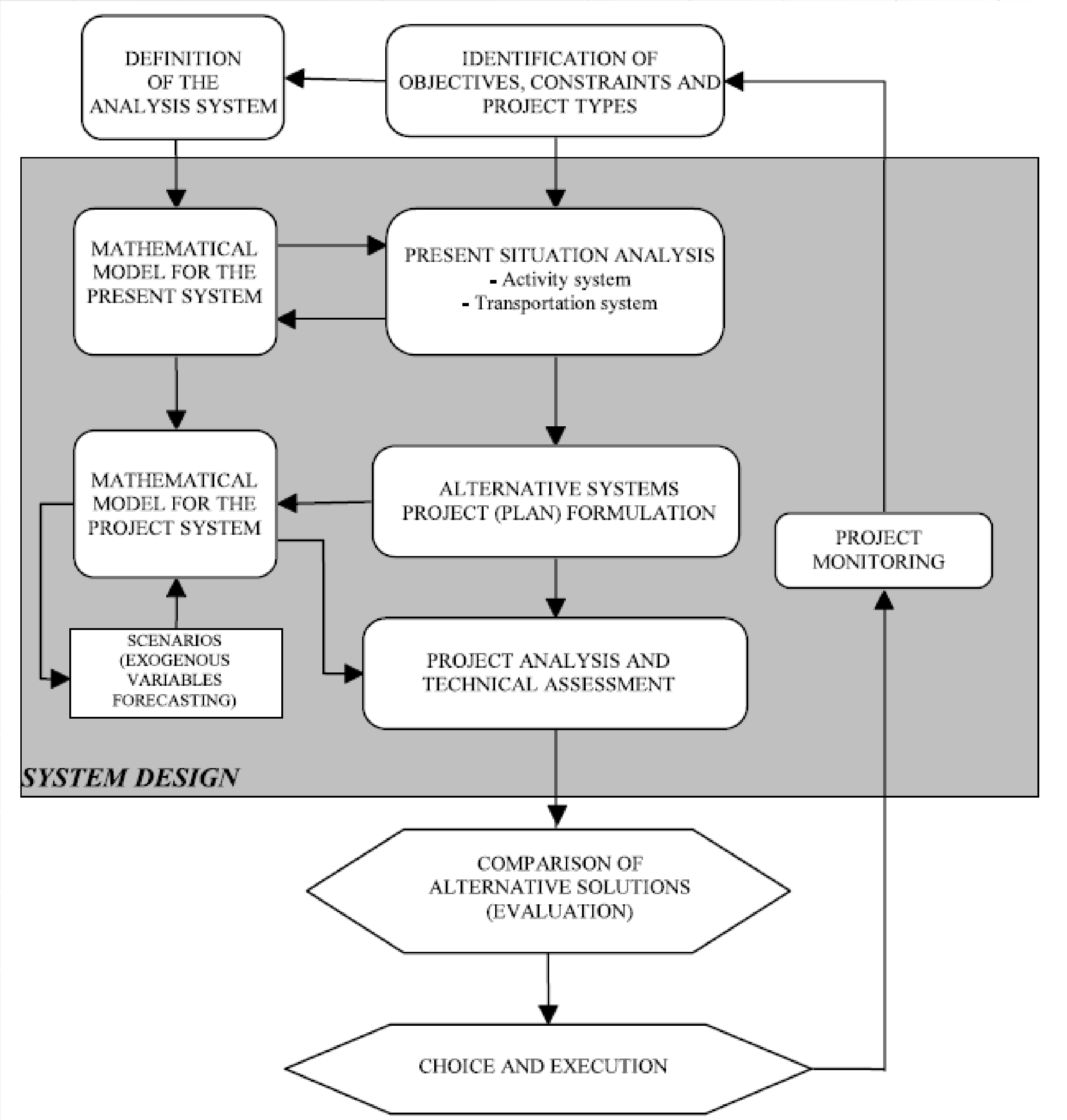
- **The aim of transportation** systems engineering, is to design transportation systems using **quantitative methods**.

- If the problem at hand is long-term planning of the whole urban transportation system, including the construction of new motorways, railway lines, parking facilities, and the like, the analysis has to include the entire multimode transportation system and possibly its relationships with the urban activity system,

- The problem can be generalized to areas of different size (a region, a whole country, etc.) and extended to cover freight transportation.

Transportation systems design and the planning process

- ***objectives and constraints identification*** **-** define the type of actions that can be included in the project.
- ***analysis of the present situation*** – data on the transportation and activity systems are collected (input data for the models (supply, demand, land use). Finding critical points
  - ***mathematical model of the present system-*** models often provide some system performance indicators (e.g., flows, saturation levels, generalized transportation costs by the O-D pair)
- ***formulation of system projects -*** sets of complementary and/or integrated actions that are internally consistent and technically feasible.
  - ***prediction of the relevant impacts*** *(required before the next step!)* Most of the impacts can be forecast quantitatively using the mathematical models and their application methods. Numerous evaluation of possible scenarios.
- ***technical assessment of the projects*** - verifies that the elements of the supply system will function within their ranges of economic validity and technical feasibility



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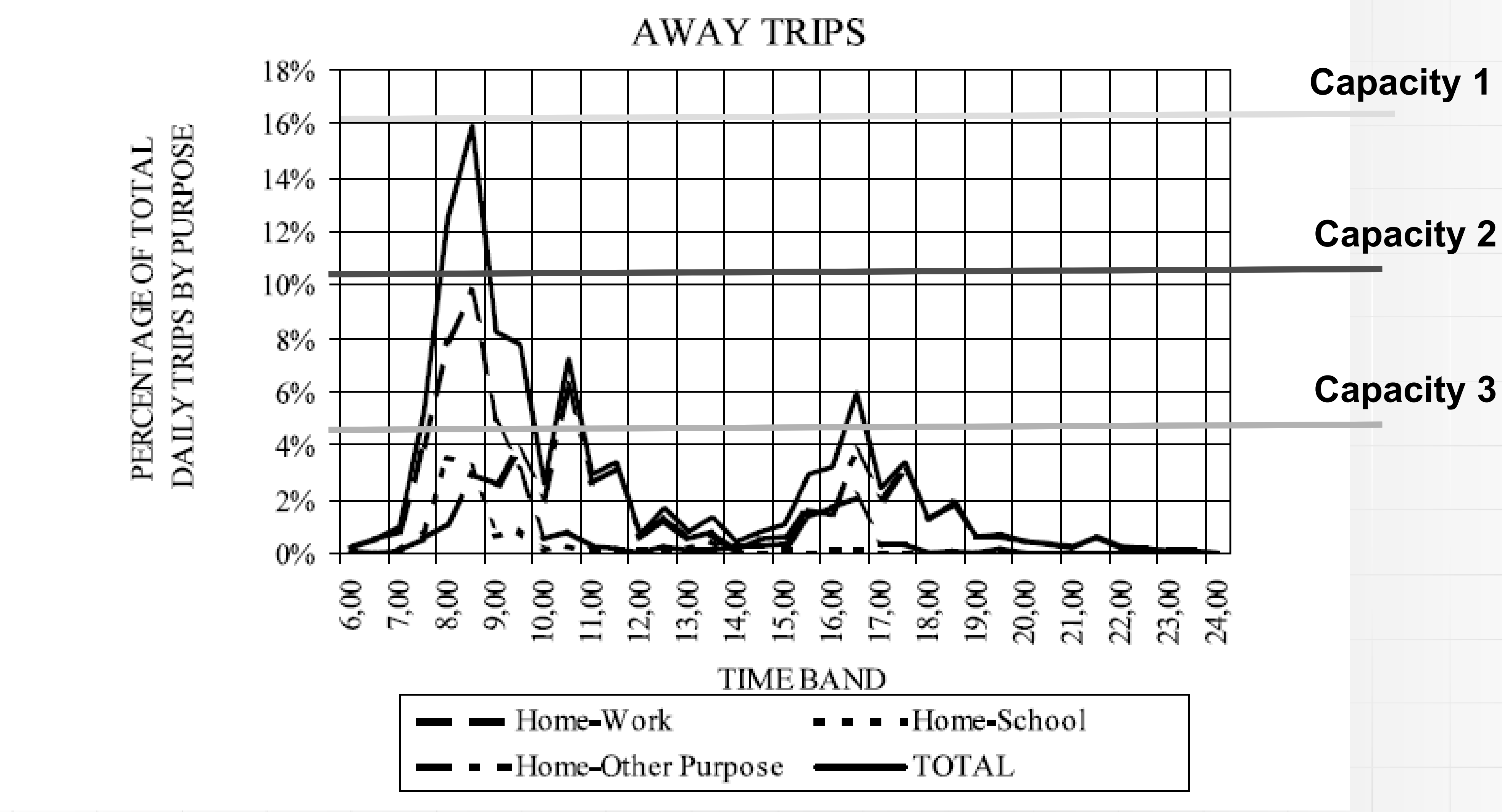
Transportation systems design;
Central challange

- **the capacity** is the maximum flow that can go through any transportation facility. Depends upon:
  - ✓ infrastructure
  - ✓ vehicles
  - ✓ technology
  - ✓ labor
  - ✓ operating policy
  - ✓ institutional factors
  - ✓ external factors (e.g., “clean air”, safety, regulation)
- **Cost/level-of-service** trade-offs are a fundamental tension for the transportation provider and for the transportation customer, as well as between them

If we underinvest in capacity, our level-of-service may be uncompetitive. If we overinvest, level-of-service may be fine, but costs will be high and our prices may not be competitive

Capacity Decisions



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Transportation system engineering

- **Identification of relevant spatial dimensions**
  - *Definition of the study area*

  trips of interest should have their origin and destination inside the study area. The limit of the study area is the *area boundary*. Outside this boundary is the external area, which is only con sidered through its connections with the analysis system.

  - **Subdivision of the area into traffic zones (zoning)**

  traffic analysis zones (TAZs).
  interzonal trips, intrazonal trips

  - **Identification of the basic network**

- **Definition of relevant components of travel demand**

The number of trips undertaken in an urban area and the frequency of transit services vary by time of day, by day of the week, and so on.

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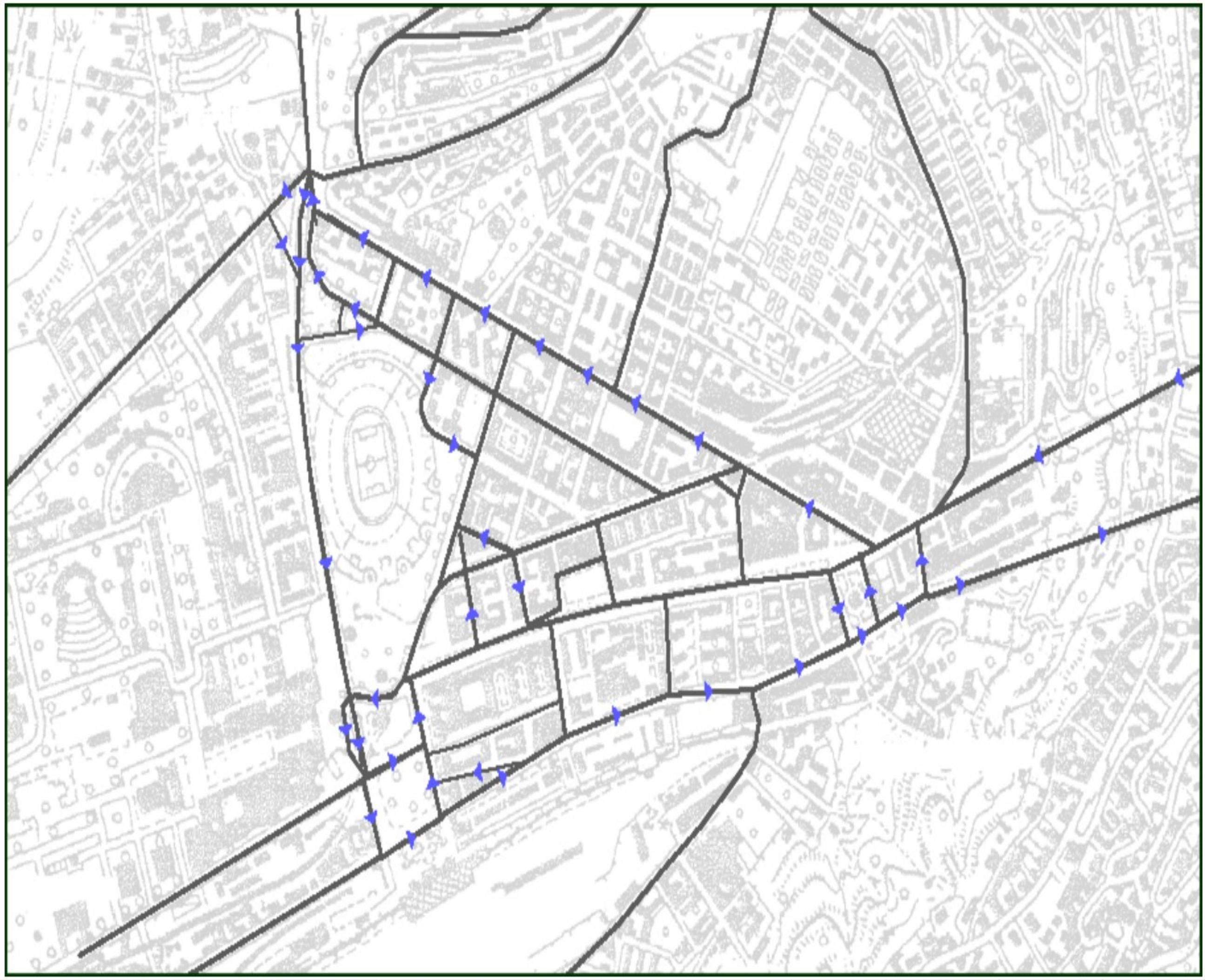
Transportation system engineering;

Basic Network

- **Physical geographic separators** (e.g., rivers, railway lines, etc.);

- **Traffic zones are often defined as aggregations** of official administrative areas(e.g., census geographic units, municipalities, or provinces). This allows each zone to be associated with the statistical data;

- **A different level of zoning detail** may be adopted for different parts of the study area depending on the precision needed;
- **A traffic zone should group connected** portions of the study area that are relatively homogeneous with respect both to their land use (e.g., residential or commercial uses in urban areas; industrial or rural uses in outlying areas) and to their accessibility to transportation facilities and services.



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Mathematical models
O-D matrices

Matrix entry d_{od} gives the number of trips made in the reference period from origin zone *o* to destination zone *d* (the *O-D flow*)

total number of trips leaving zone *o* in flow - ***produced* or *generated* by zone *o***.

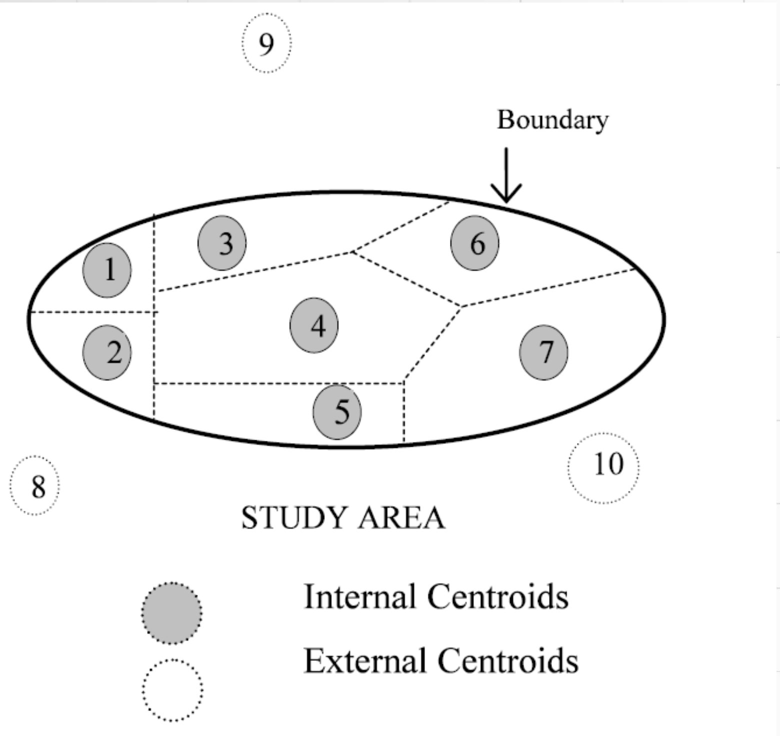
$$d_{o\cdot} = \sum_d d_{od}$$

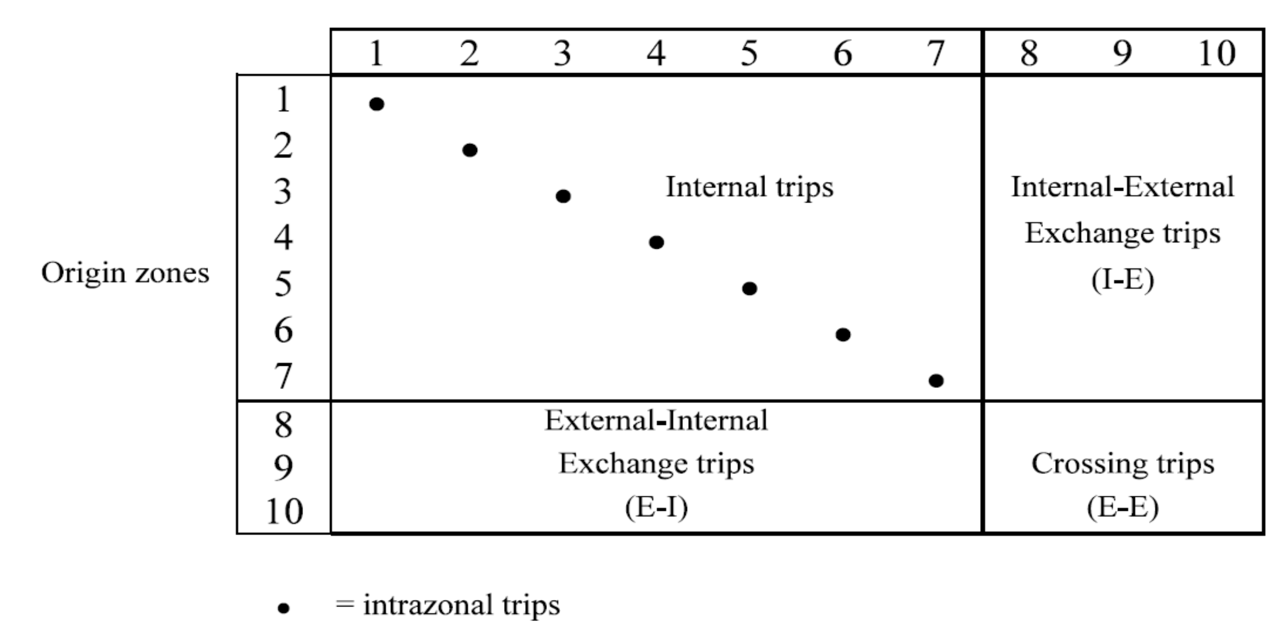
the number of trips arriving in zone *d* in the reference period - **the flow *attracted* by zone *d***

$$d_{\cdot d} = \sum_o d_{od}$$

The total number of trips made in the study area in the reference interval is indicated by *d..*:

$$d_{\cdot\cdot} = \sum_o \sum_d d_{od}$$





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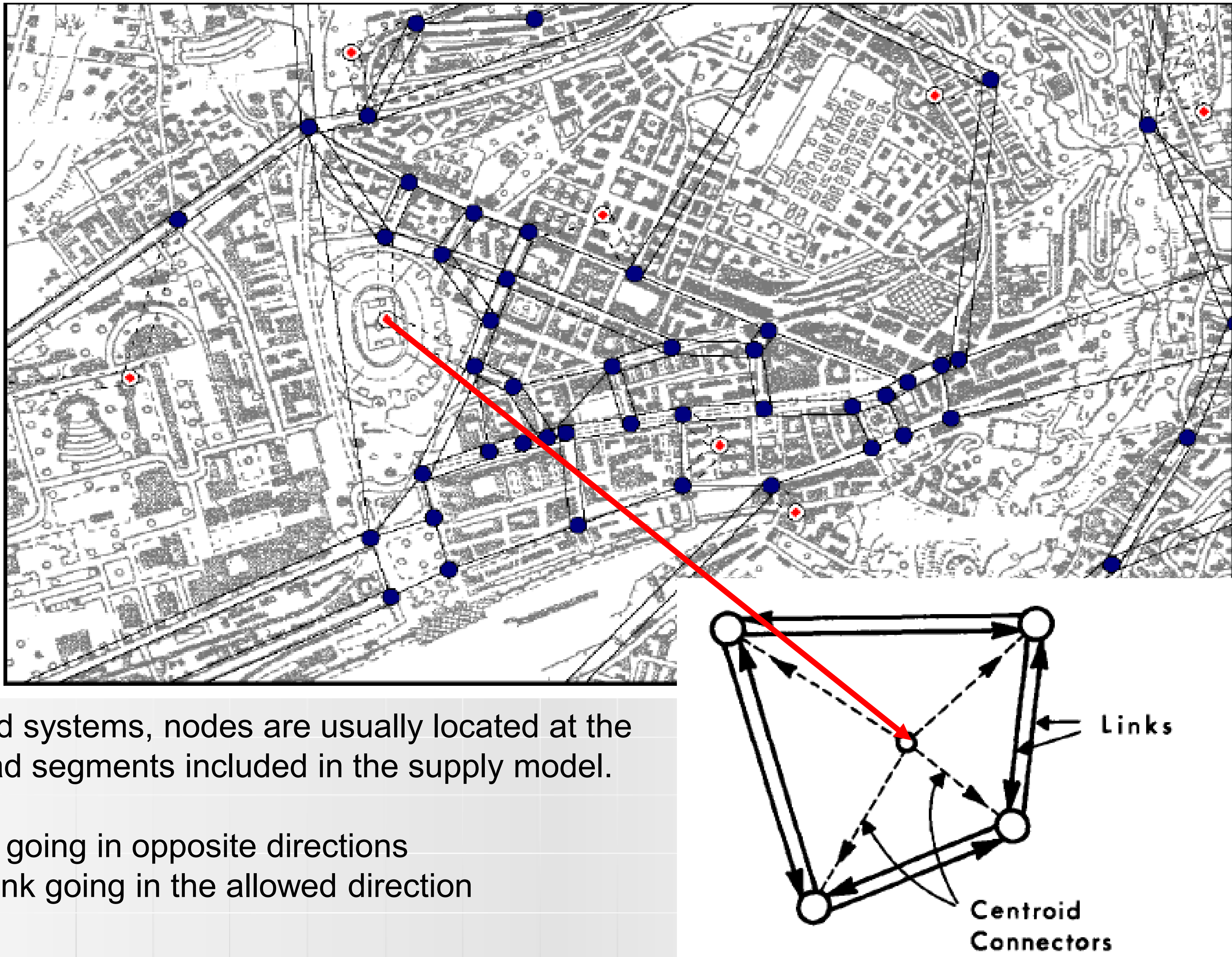
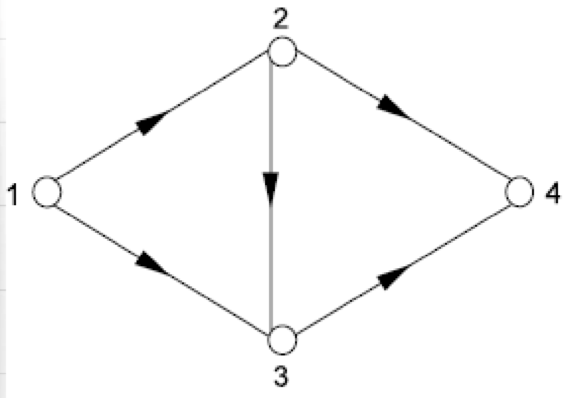
Graph models ;



In graphs representing road systems, nodes are usually located at the intersections between road segments included in the supply model.

- **two-way road** two links going in opposite directions
- **one-way road** a single link going in the allowed direction

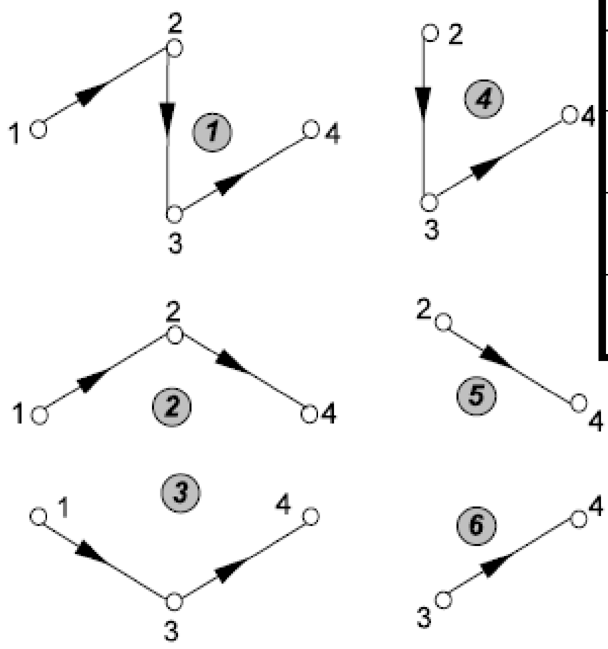
GRAPH



$G \equiv (N, L)$
$N \equiv \{(1, 2, 3, 4)\}$
$L \equiv \{(1,2), (1,3), (2,3), (2,4), (3,4)\}$

Origin centroid nodes 1, 2, 3
Destination centroid nodes 4

PATHS



O-D Pairs		1-4			2-4		3-4
Links \ Paths		1	2	3	4	5	6
1		1	1	0	0	0	0
2		0	0	1	0	0	0
3		1	0	0	1	0	0
4		0	1	0	0	1	0
5		1	0	1	1	0	1

Link flow vector   *Path flow vector*   *Link-path incidence matrix*

$$\boldsymbol{f} = \begin{bmatrix} f_{12} \\ f_{13} \\ f_{23} \\ f_{24} \\ f_{34} \end{bmatrix} \qquad \boldsymbol{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{bmatrix} \qquad \Delta = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1,2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1,3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2,3 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2,4 & 0 & 1 & 0 & 0 & 1 & 0 \\ 3,4 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}$$

Link flow vector

$$\boldsymbol{f} = \begin{bmatrix} f_{12} \\ f_{13} \\ f_{23} \\ f_{24} \\ f_{34} \end{bmatrix} = \Delta \boldsymbol{h} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix} = \begin{bmatrix} h_1 + h_2 \\ h_3 \\ h_1 + h_4 \\ h_2 + h_5 \\ h_1 + h_3 + h_4 + h_6 \end{bmatrix}$$

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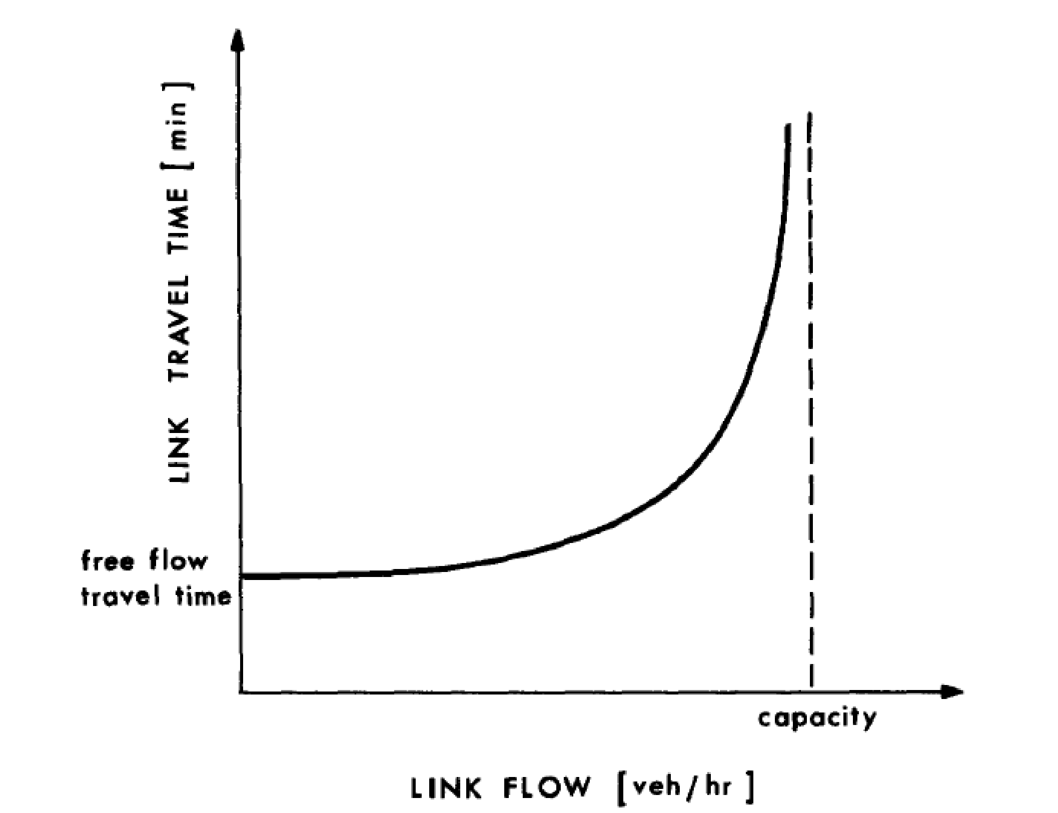
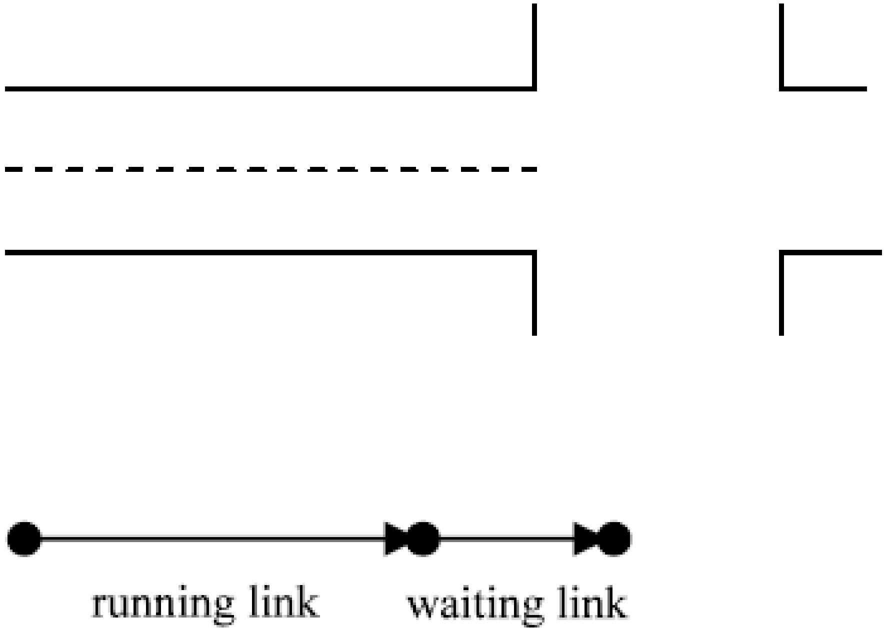
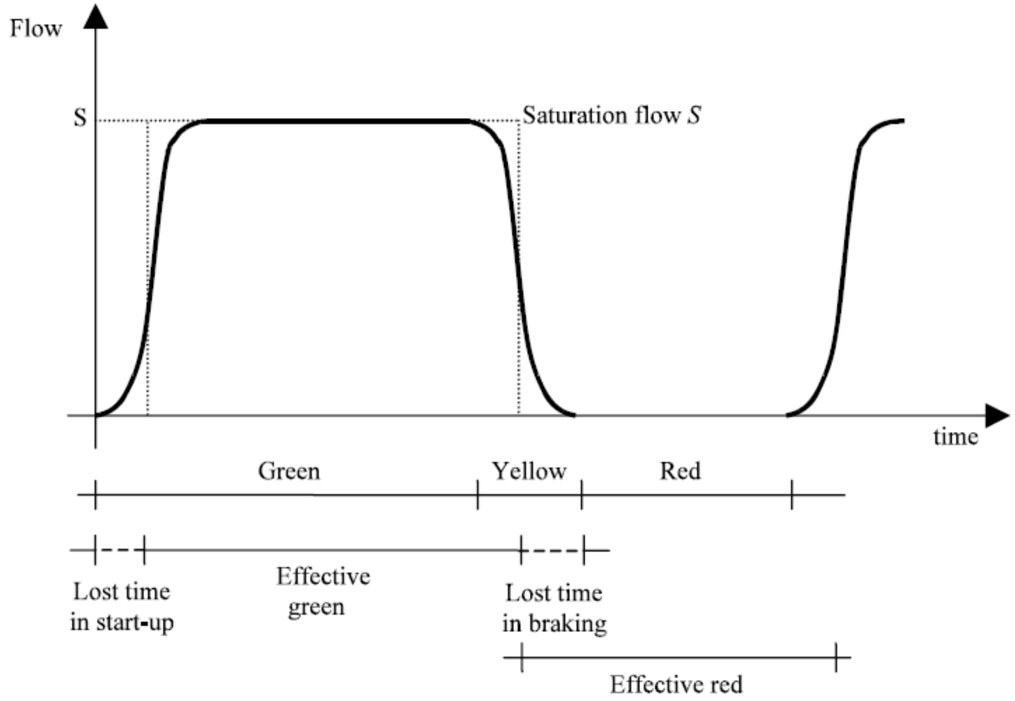
Model accuracy

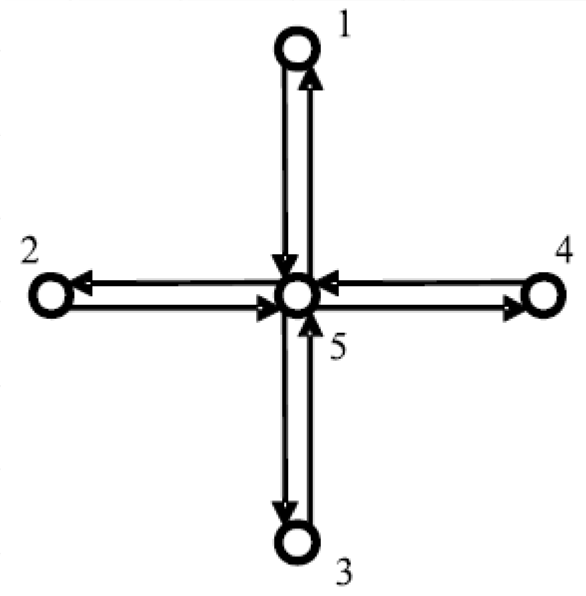
running links - the vehicle's real movement as the trip along a motorway or urban road
section; and
***waiting* or *queuing links*,** queuing at intersections, toll barier, etc.

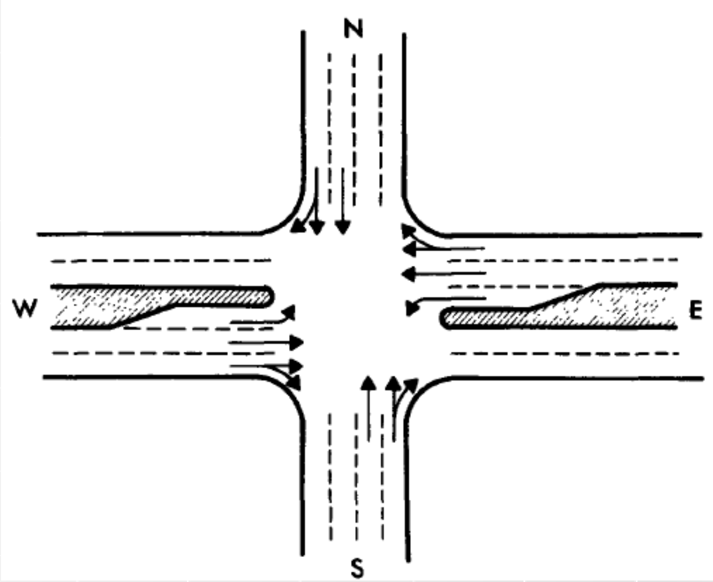


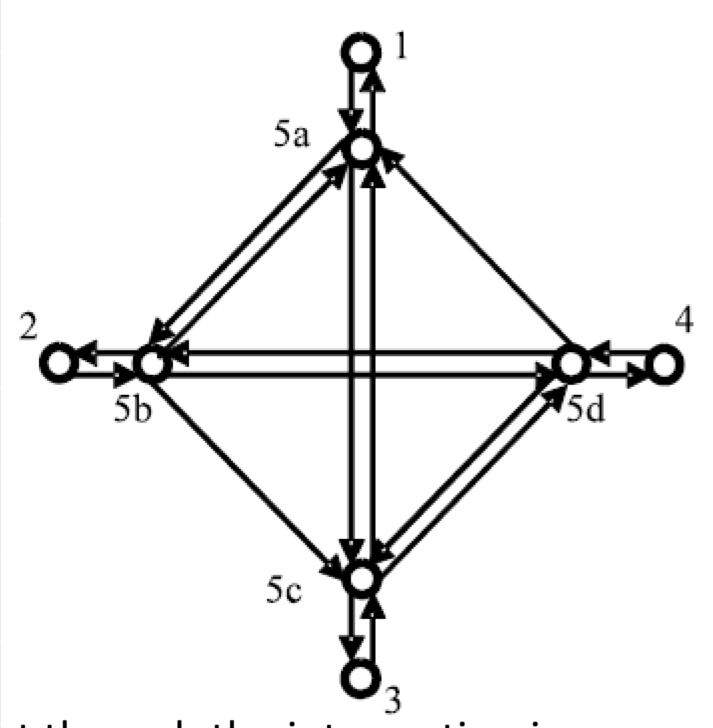




Model accuracy



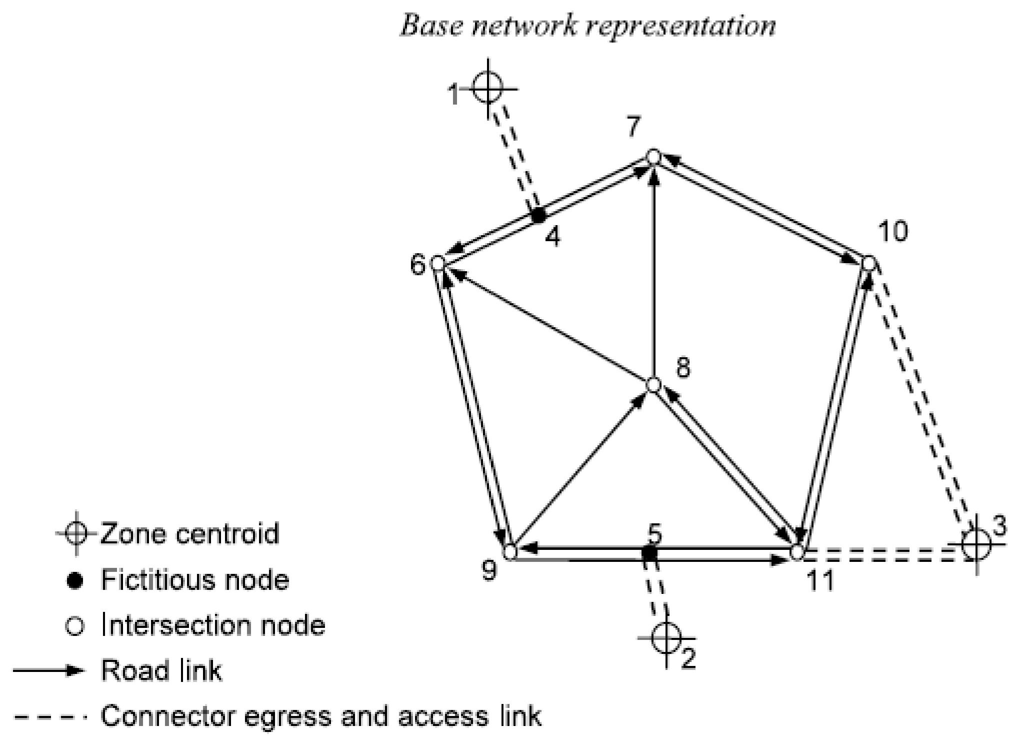


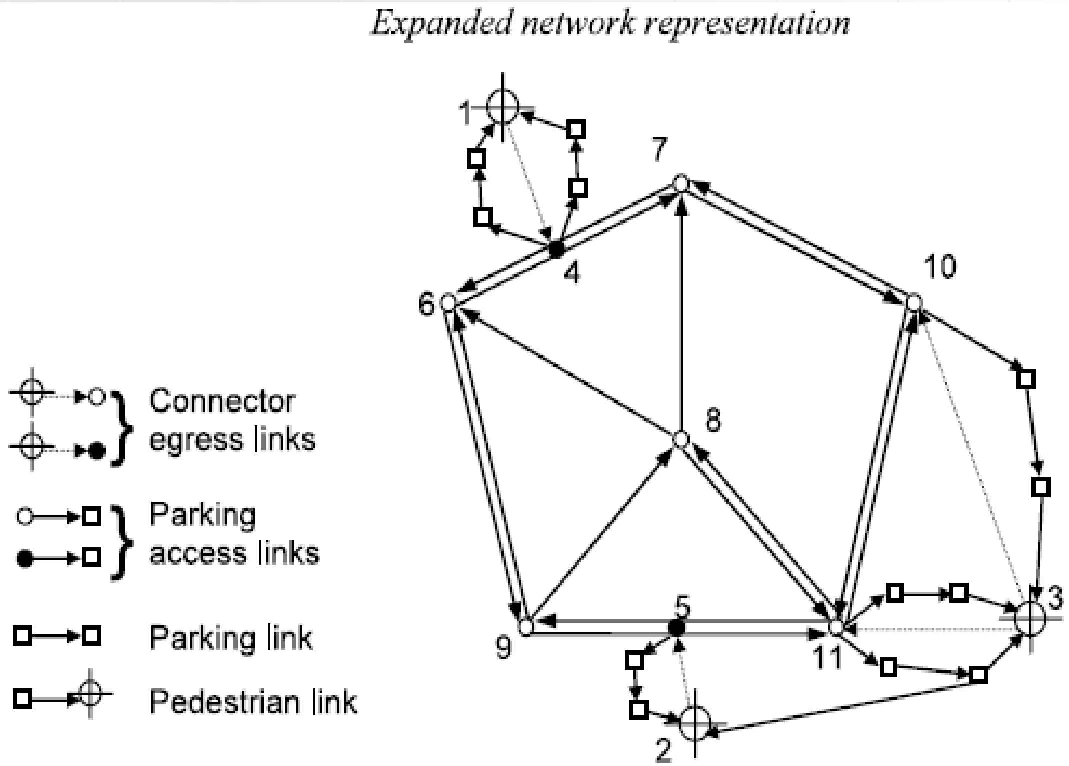


- Cannot be used to represent turning restrictions.
- it assumes that all flow entering the intersection from a particular direction will experience the same travel time, regardless of where it is destined.

- Each permissible movement through the intersection is represented by a separate link.
- each intersection movement can be associated with the appropriate delay.
  - e.g. link 5d-5c left-turning delay that would be experienced by the traffic turning from the eastern approach into the southern one.
  - link 5d-5a right-turning maneuvers for flow from the east to the north, with the appropriate delay

Model accuracy; Parkings





congestion and different parking policies cannot be simulated

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Network Flow Propagation (NFP)

- In the macroscopic approach the most important factors of traffic are
- **Links flow f_a** -the average number of homogeneous units using link a (i.e., carrying out the trip phase represented by the link) in a time unit.
  - ***User flows*** relate to users, such as travelers or goods, possibly of different classes
  - ***Vehicle flows*** relate to the number of vehicles, perhaps of different types such as automobiles, buses, trains,
  - ***path flow h_k*** - *the* average number of users, who in a time unit follow path
  - **link– path incidence matrix -** the flow on each link *a* can be obtained as the sum of the flows on the various paths containing that link. This relationship can be expressed by using the elements $\boldsymbol{\delta_{ak}}$

$$f_a = \sum_i w_i f_a^i$$

$$h_k = \sum_i w_i \cdot h_k^i$$

$$f_a = \sum_k \delta_{ak} \cdot h_k$$

– Density- amount of vehicle on the road

- Capacity - that is, a maximum number of units that may use them in a given time interval.
- Max flow speed - value of flow (except the capacity Q) may occur under two different conditions
  1. **Stable** - high speed and low density
  2. **Unstable** - low speed and high density

Greenshield's model

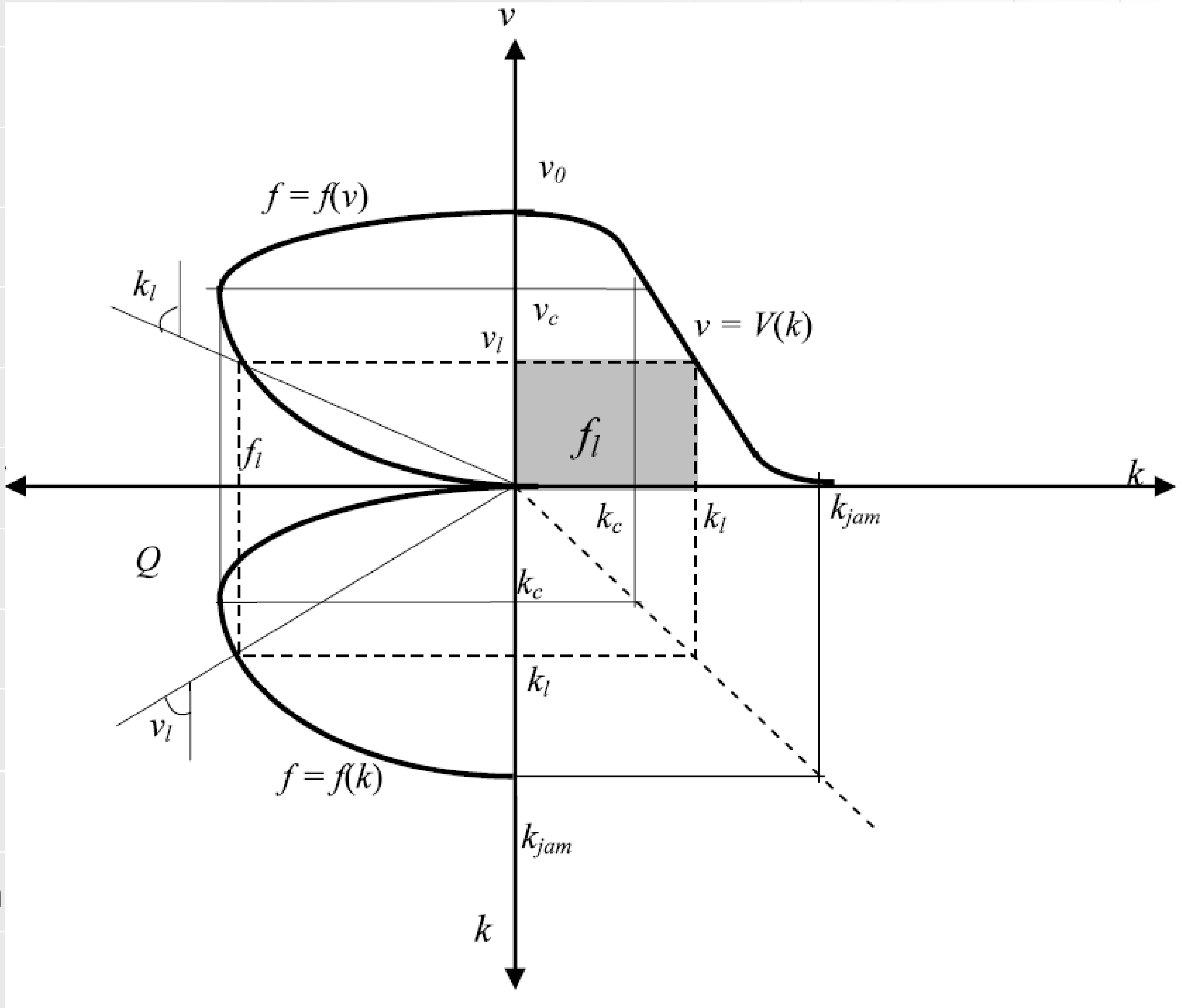
$$v_{\text{stable}}(f) = \frac{v_0}{2}\left(1 + \sqrt{1 - 4f/(v_0 k_{\text{jam}})}\right) = \frac{v_0}{2}\left(1 + \sqrt{1 - f/Q}\right)$$

$$v_{\text{unstable}}(f) = \frac{v_0}{2}\left(1 - \sqrt{1 - f/Q}\right)$$

flow may be zero under two conditions:

1. when density is zero (no vehicles on the road) - ***free-flow speed*** $\boldsymbol{v_0}$- speed assumes the theoretical maximum value,
2. when speed is zero (vehicles are not moving) ***jam density***, $\boldsymbol{k_{jam}}$density assumes the theoretical maximum value

- The peak of the *speed–flow* (and *density–flow*) curve occurs at the theoretical maximum flow, *capacity Q* of the facility; the corresponding **speed** $\boldsymbol{v_c}$ and **density** $\boldsymbol{k_c}$ are referred to as the critical
- Thus any value of flow (except the capacity) may occur under two different conditions: low speed and high density and high speed and low density.
    - **Increase in density** will cause a decrease in speed and thus in flow
    - **increase in density** will cause a decrease in speed and an increase in flow.
- At capacity (or at critical speed or density) the stream is nonstable, this being a boundary condition between the other two.



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All this have impact on travel time and costs

Generalized transportation cost

- the cost function can be obtained as the sum of three performance functions:

$$c_a(f) = \beta_1 tr_a(f) + \beta_2 tw_a(f) + \beta_3 mc_a(f)$$

- In the most general case, the monetary cost term mc_a includes the cost items that are perceived by the user.
  - motor oil,
  - tires,
  - Toll
  - Fuel consumption

$$mc_a = mc_{\text{toll}} + mc_{\text{fuel}}(f)$$

- Time of travel – depends upon the ling types (running ling and waiting links).

  - Running links –
    - motorways links.
    - urban,
    - extra-urban

$$tr_a = L_a / v_a(f_a)$$

tr_a	is the running time on link a
f_a	is the flow on link a
L_a	is the length of the running link a
v_a	is the mean speed on link a assuming a stable regime

  - Waiting lings (queuing theory) (tw_a)

Travel time for extraurban Road Links

- Users traveling on an extraurban road behave differently according to the number of lanes available for each direction.
- the capacity and travel conditions in each direction are not influenced by the flow in the opposite direction
- In the case of roads with one lane in each direction, link performances depend on the flow in both directions:

In practice, it is often assumed that link capacity has a value common to both directions,

$$tr_a(f_a, f_{a^*}) = \frac{L_a}{v_{0a}} + \gamma_a \left(\frac{L_a}{v_{ca}} - \frac{L_a}{v_{0a}} \right) \left(\frac{f_a + f_{a^*}}{Q_{aa^*}} \right)^{\gamma_2}$$

a^* , Q^*_{aa} are opposite direction is denoted lanes, overall capacity in both directions

In an urban context, given the relatively short lengths of road sections, travel speed is more dependent upon road physical and functional characteristics than upon the flow traveling on them. **The higher the dependence is on factors such as section bendiness or roadside parking, the lower the impact of flow**

Travel time for motorway links

- On motorway links flow conditions are typically uninterrupted and it is assumed that the waiting time component is negligible because it occurs on those sections (ramps, tollbooths, etc.) that are usually represented by different links.

$$tr_a(f_a) = \frac{L_a}{v_{oa}} + \left(\frac{L_a}{v_{ca}} - \frac{L_a}{v_{oa}}\right)\left(\frac{f_a}{Q_a}\right)^4$$

$\boldsymbol{L_a}$ is the length of link *a*
$\boldsymbol{v_{0a}}$ is the free-flow average speed
$\boldsymbol{v_{ca}}$ is the average speed with flow equal to capacity
$\boldsymbol{Q_a}$ is link capacity, that is, the average maximum number of equivalent vehicles that can travel along the road section in a time unit. Capacity is usually obtained as the product of the number of lanes on the link *a*, N_a, and lane capacity, Q_{ua}

Waiting links

- In order to avoid unrealistic waiting times and for reasons of theoretical and computational convenience, two different methods can be adopted
- The first, and less precise, method assumes that understurated waiting time holds for flow values up to a fraction α of the capacity, for example, $f_a \leq 0.95Q$

$$tw_a(f_a) = tw_a(\alpha Q_a) + K(f_a - \alpha Q_a)$$

$$K = \frac{T_s^2 + \sigma_s^2}{2} \cdot \frac{1}{(1-\alpha)^2}$$

- The deterministic average (oversaturation) delay is:

$$tw_a^d = T_s + \left(\frac{f_a}{Q_a} - 1\right)\frac{T}{2}$$

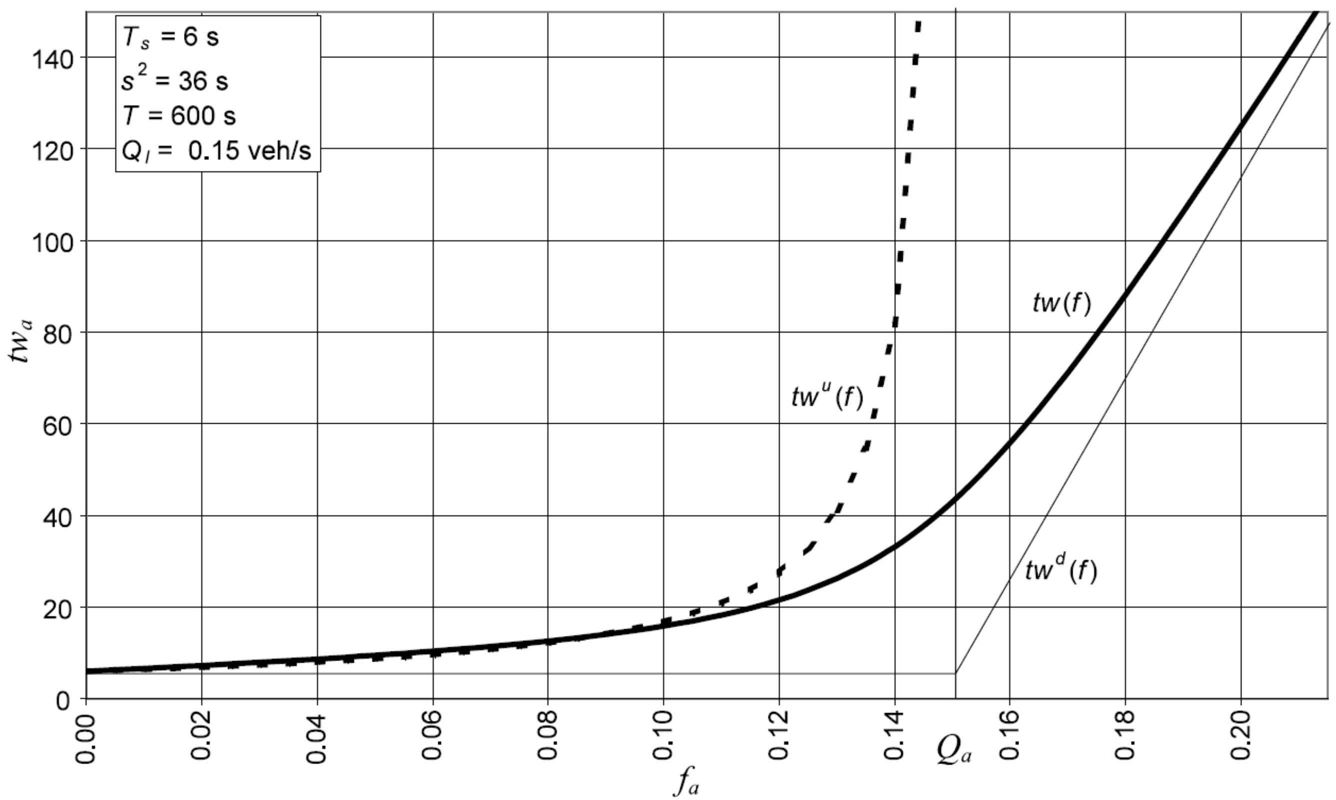
Waiting links

The average delay tw_a can be calculated by combining the stochastic undersaturation average delay tw_{ua}

$$tw_a^u(f_a) = T_s + (T_s^2 + \sigma_s^2) \cdot \frac{f_a}{2} \cdot \frac{1}{1 - f_a/Q_a}$$

with the deterministic average oversaturation delay tw_{da}, expressed by

$$tw_a^d = T_s + \left(\frac{f_a}{Q_a} - 1\right)\frac{T}{2}$$



The combined delay function is such that the deterministic delay function is its oblique asymptote. The following equation results

$$tw_a(f_a) = T_s + (T_s^2 + \sigma^2)\frac{f_a}{2} + \frac{T}{4}\left\{\frac{f_a}{Q_a} - 1 + \left[\left(\frac{f_a}{Q} - 1\right)^2 + \frac{4(f_a/Q_a)}{Q_a T}\right]^{1/2}\right\}$$

Waiting lings ;
Queuing model

- **Overall Delay Models.** The total (mean individual) delay equals the sum of the deterministic and the stochastic terms and sometimes, of terms calibrated through experimental observations
- One of the best known expressions **is Webster's two-term formula,**

$$tw_a(f_a) = 0.9\left[\frac{T_c(1-\mu)^2}{2(1-f_a/S_a)} + \frac{(f_a/Q_a)^2}{2f_a(1-f_a/Q_a)}\right]$$

Tc is the cycle length
μ is the effective green to cycle length ratio for the lane group represented by link *a*
Qa is the capacity of the lane group represented by link *a*

Matematical programs;
Large networks

- Graphical methods cannot be used to solve for equilibrium over networks with a large number of nodes, links, and O-D pairs;
- The mathematical approach involves the formulation of a mathematical program, the solution of which is the user-equilibrium flow pattern;
- It is used often in operations research, in cases in which it is easier to minimize the equivalent;
- The basic problem is to find the link flows given the origin-destination trip rates the network, and the link performance functions (minimisation of the travel time);
- the travel time on a given link is a function of the flow on that link only and not of the flow on any other link in the network.
- **The equilibrium assignment problem is to find the link flows, x, that satisfy the user-equilibrium criterion when all the origin-destination entries, q, have been appropriately assigned;**

$$min\ z(x) = \sum_a \int_0^{x_a} t_a(\omega) d\omega$$

The volume-delay curves for the two links are given by

$$t_1 = 2 + x_1$$
$$t_2 = 1 + 2x_2$$

The trip rate **q** is 5 units of flow, (i.e. trafick flow on each roads) is given by

$$q = x_1 + x_2 = 2 + 3 = 5$$

Equiribliom conditions exists for

$$t_1 \le t_2 \; if \; x_1 > 0$$
$$t_1 \ge t_2 \; if \; x_2 > 0$$

For this example it can be verified by inspection that both paths will be used at equilibrium and the last equation can therefore be written simply - given that $x_1 > 0$ and $x_2 > 0$) as $\boldsymbol{t_1 = t_2}$ (user equilibrion condition)

With this equilibrium condition we can solve the equilibrium problem by solving the sets of equation explained above

$$x_1 = 3; x_2 = 2; t_1 = t_2 = 5$$

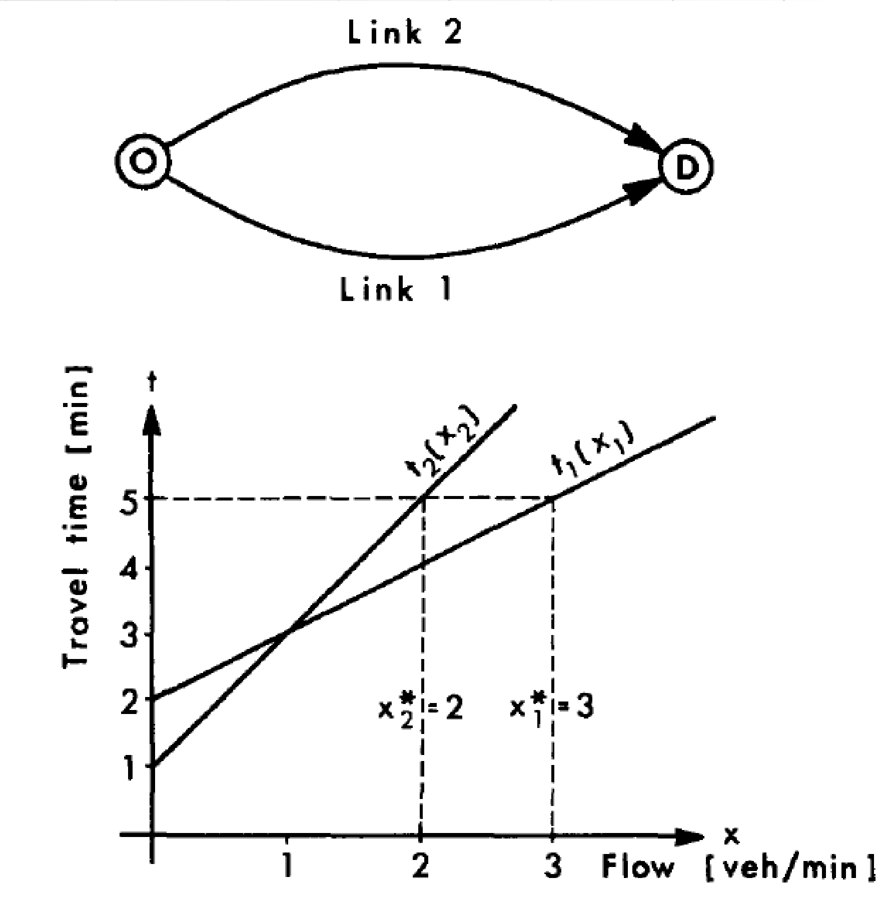
The equilibrium assignment problem is to find the link flows, x, that satisfy the user-equilibrium criterion when all the origin-destination entries, q, have been appropriately assigned;

$$\boldsymbol{min\, z(x) = \sum_a \int_0^{x_a} t_a(\omega) d\omega}$$

$$min z\, z(x) = \int_0^{x_1} (2+\omega) d\omega + \int_0^{x_2} (1+2\omega) d\omega = \int_0^{x_1} (2+\omega) d\omega + \int_0^{5-x_1} (1+2\omega) d\omega$$

If the solution satisfies the constraints, it is valid for the constrained program as well

$$\boldsymbol{z(x_1) = 1{,}5x_1^2 - 9x_1 + 30}$$



SHEFFI, Y. 1984. *Urban Transportation Networks: Equilibrium Analysis With Mathematical Programming Methods*, Prentice-Hall.

This function attains its minimum at $x_1^* = 3$ and $x_2^* = 3$

The solution of the mathematical program is identical to the solution of the equilibrium equations.

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Transportation systems design;

Equilibrium

- The classical view of a (perfectly competitive) economic market for a certain good includes two interacting groups: **the producers and the consumers**. The behavior of the **producers is characterized by a supply function** and the behavior of the **consumers is characterized by a demand function.**

- The point where the two curves intersect is characterized by the "**market clearing**" price, P*, and **quantity produced**, Q*.

- If the price is higher than P*, production will be higher than consumption,

- If the price is lower than P*, the quantity demanded is higher than the production.

- Such a situation is again unstable since producers will try to increase prices in order to capture the consumers' willingness to pay more. Such price increases will lead to higher production and lower demand. Thus, if the prices are either lower or higher than P*, market forces will tend to push the price toward its "market clearing" level.

- At the equilibrium the price will be stable and thus the point *(P*,* Q*) is known as the *equilibrium* point.

- **The equilibrium in the urban transportation market is necessarily reached over the urban network of streets and transit lines.**