Basic od vehicles dynamics

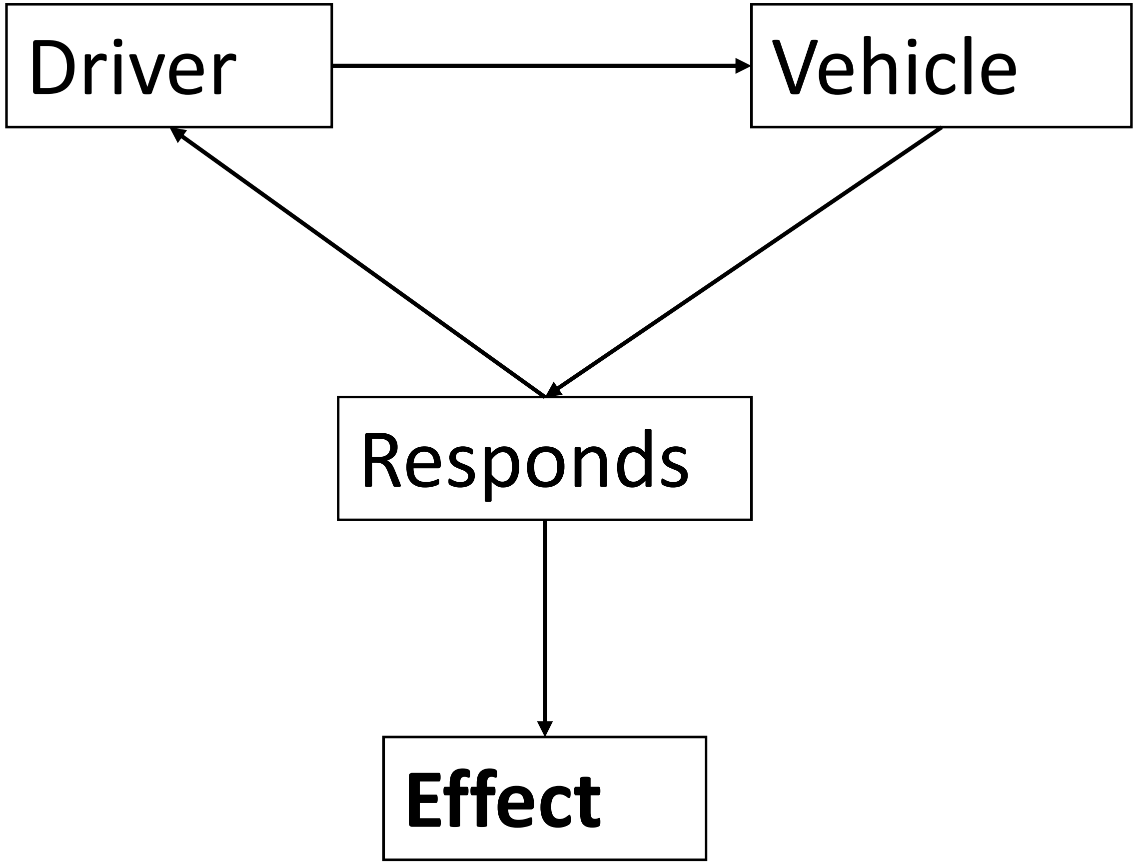
Motion resistance

Basic od vehicles dynamics

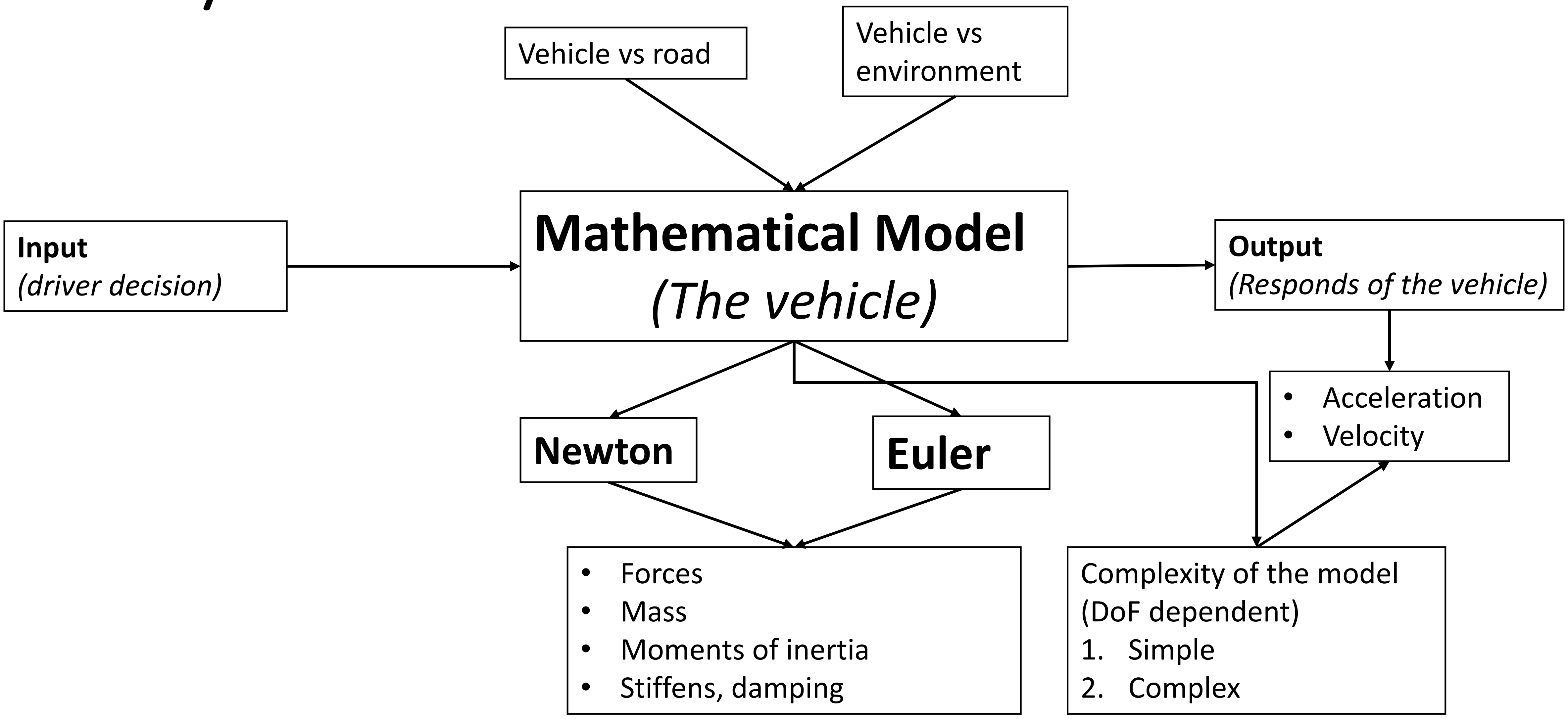
Wrocław
University
of Science
and Technology

Vehicle dynamics

1. **Vehicle – driver – road interaction**
2. **Vehicle safety** (not crashworthiness – **active safety**)
3. **Comfort of driving**
4. **Economics**

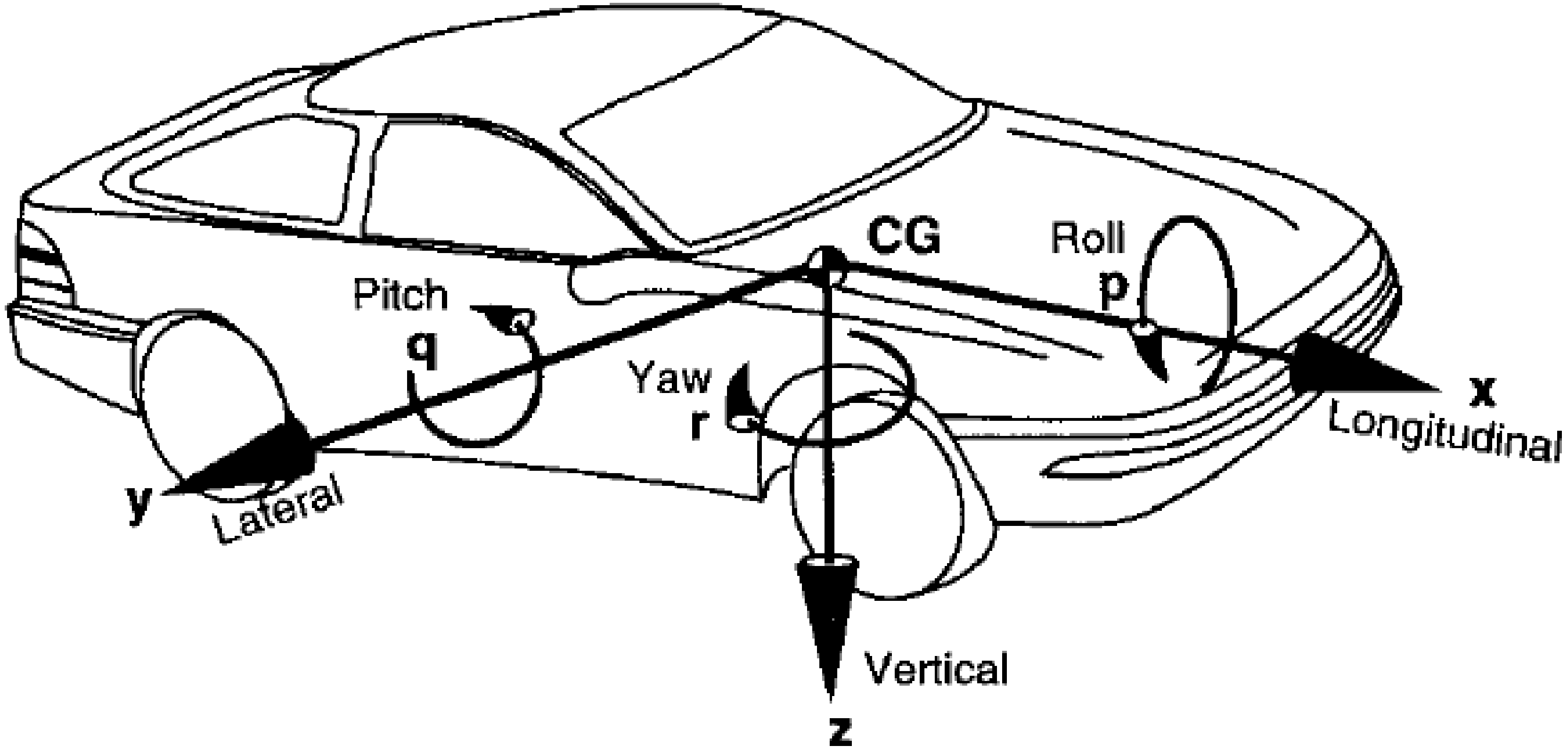


Vehicle dynamics



Vehicle dynamics
model complexity

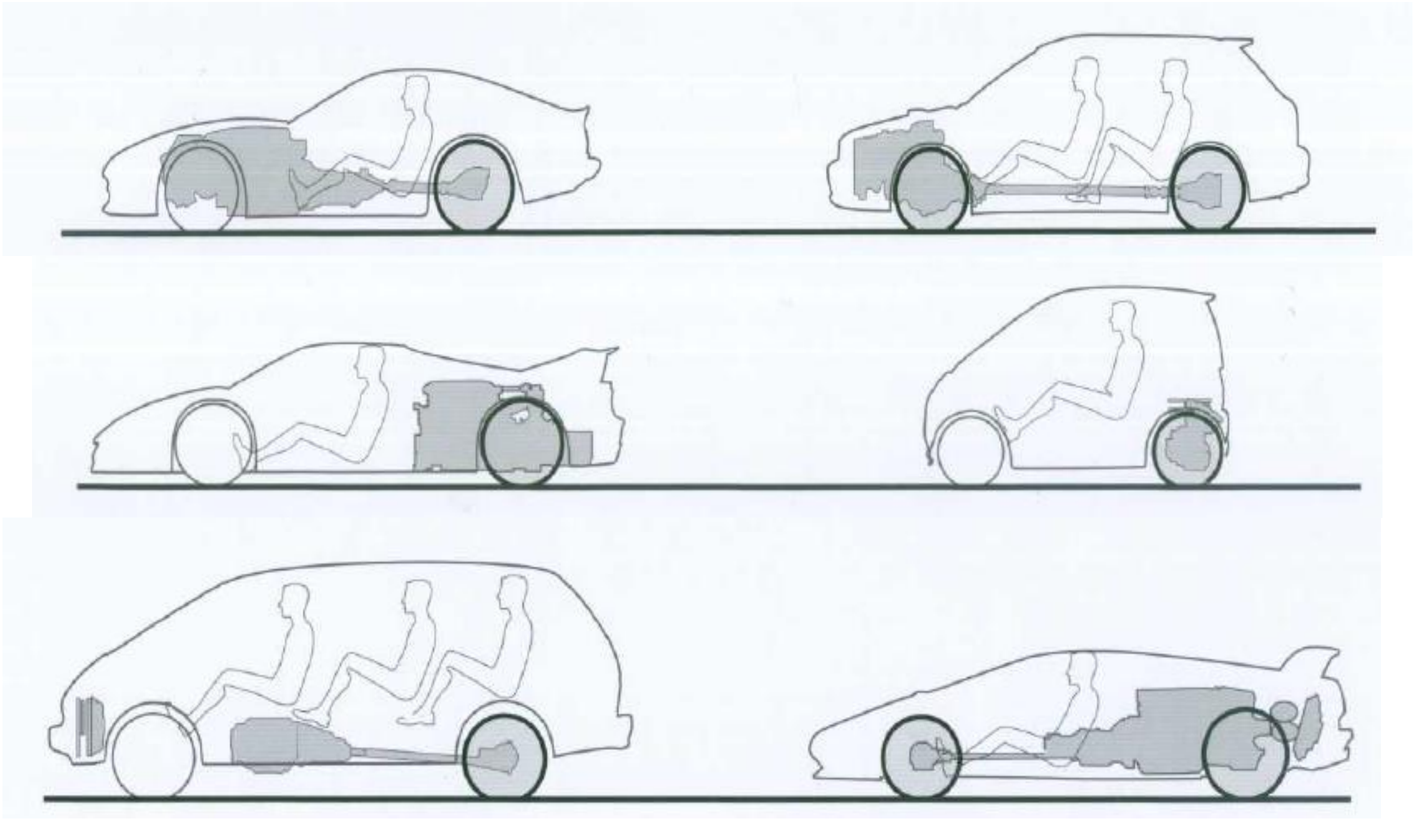
- **Longitudinal**
- **Lateral**
- **Vertical**
- Longitudinal vs vertical
- Lateral vs vertical
- Longitudinal vs lateral vs vertical



GILLESPIE, T. D. 1992. *Fundamentals of Vehicle Dynamics*, Society of Automotive Engineers.

Center of gravity
CG (CoG)

The imaginary point at which the vehicle's entire mass can be concentrated



MACEY, S., WARDLE, G. & GILLES, R. 2009. *H-point: The Fundamentals of Car Design & Packaging*, Design Studio Press.

Longitudinal dynamics

Parked vehicle (simples approach)

- No movement – no acceleration
- **CG determination;**
- The normal force under each axis calculations;
- **A symmetric two-axel vehicle is equivalent to a rigid beam having two supports**.

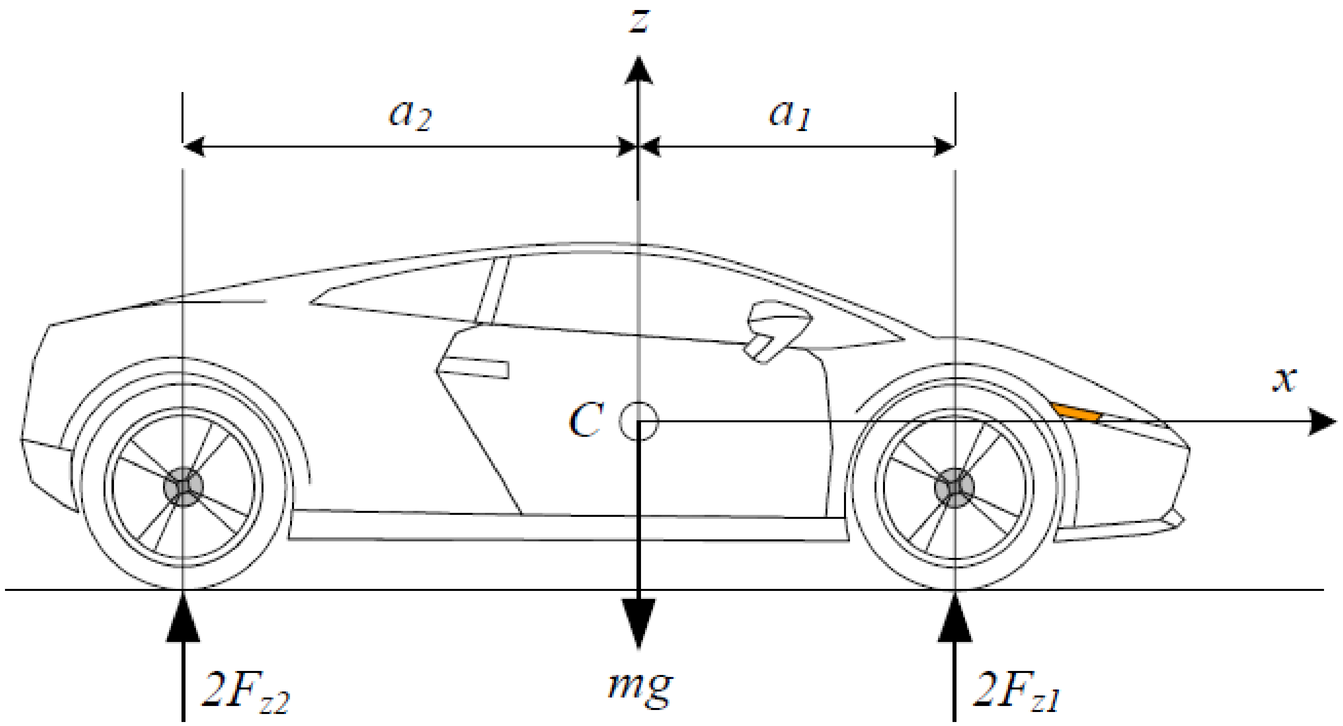
$$\sum F_z = 0$$
$$\sum M_y = 0$$

$$2F_{z_1} + 2F_{z_2} - mg = 0$$
$$-2F_{z_1} a_1 + 2F_{z_2} a_2 = 0$$

$$l = a_1 + a_2$$

$$F_{z_1} = \frac{1}{2} mg \frac{a_2}{a_1 + a_2} = \frac{1}{2} mg \frac{a_2}{l}$$

$$F_{z_2} = \frac{1}{2} mg \frac{a_1}{a_1 + a_2} = \frac{1}{2} mg \frac{a_1}{l}$$

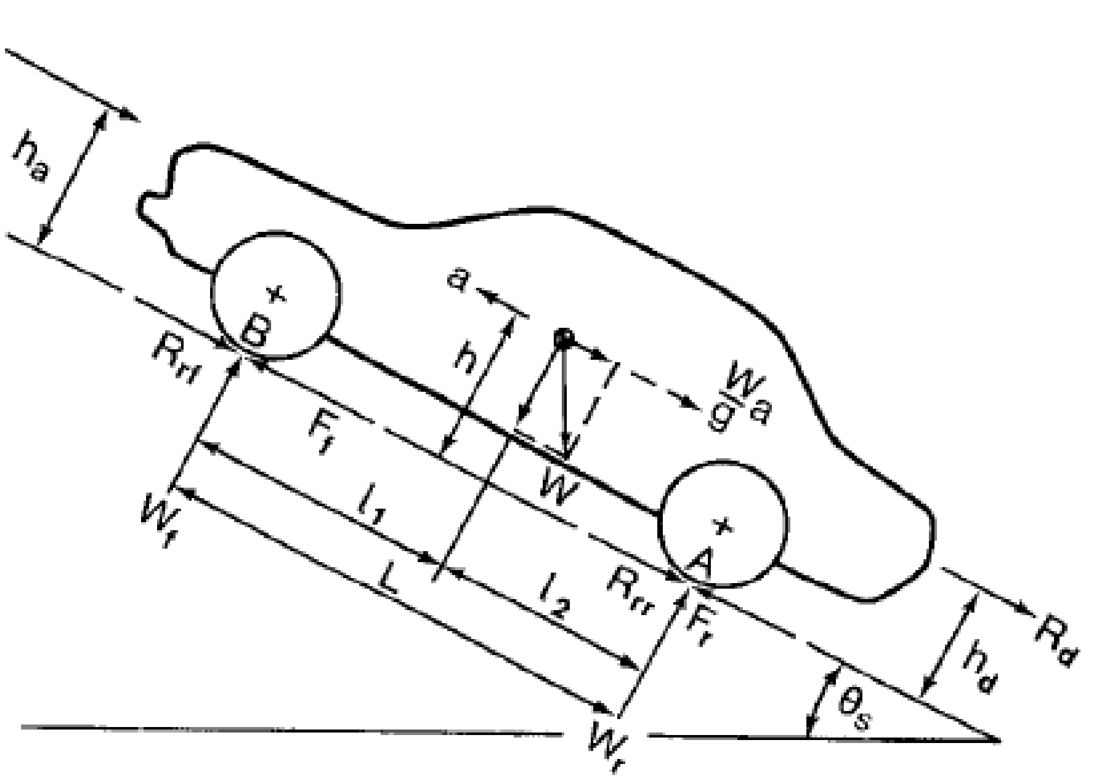


JAZAR, R. N. 2008. *Vehicle Dynamics: Theory and Application*, Springer US.

Acceleration performance

The axle loads determine the tractive effort obtainable at each axle, affecting the acceleration, gradeability, maximum speed, and drawbar effort

- $\boldsymbol{W}$ is the weight of the vehicle acting at its CG with a magnitude equal to its mass times the acceleration of gravity
- $\frac{\boldsymbol{W}}{\boldsymbol{g}\boldsymbol{a}}$ is a d' Alembert force. If the vehicle is accelerating along the road, it is convenient to represent the effect by an equivalent inertial force
- $\boldsymbol{W_f}, \boldsymbol{W_r}$ the dynamic weights carried on the front and rear wheels
- $\boldsymbol{F_f}, \boldsymbol{F_r}$ Tractive effort (for FWD $F_r = 0$, RWD $F_f = 0$)



WONG, J. Y. 2001. *Theory of Ground Vehicles*, Wiley.

Moving vehicle must be strong enough to overcome all resisting forces

- $\boldsymbol{R_{rf}}, \boldsymbol{R_{rr}}$ rolling resistance of the front and rear tires
- $\boldsymbol{R_a}$ aerodynamics resistance
- $\boldsymbol{R_d}$ drawbar load
- $\boldsymbol{R_g}(\boldsymbol{W} \cdot \boldsymbol{sin\theta})$ grade resistance

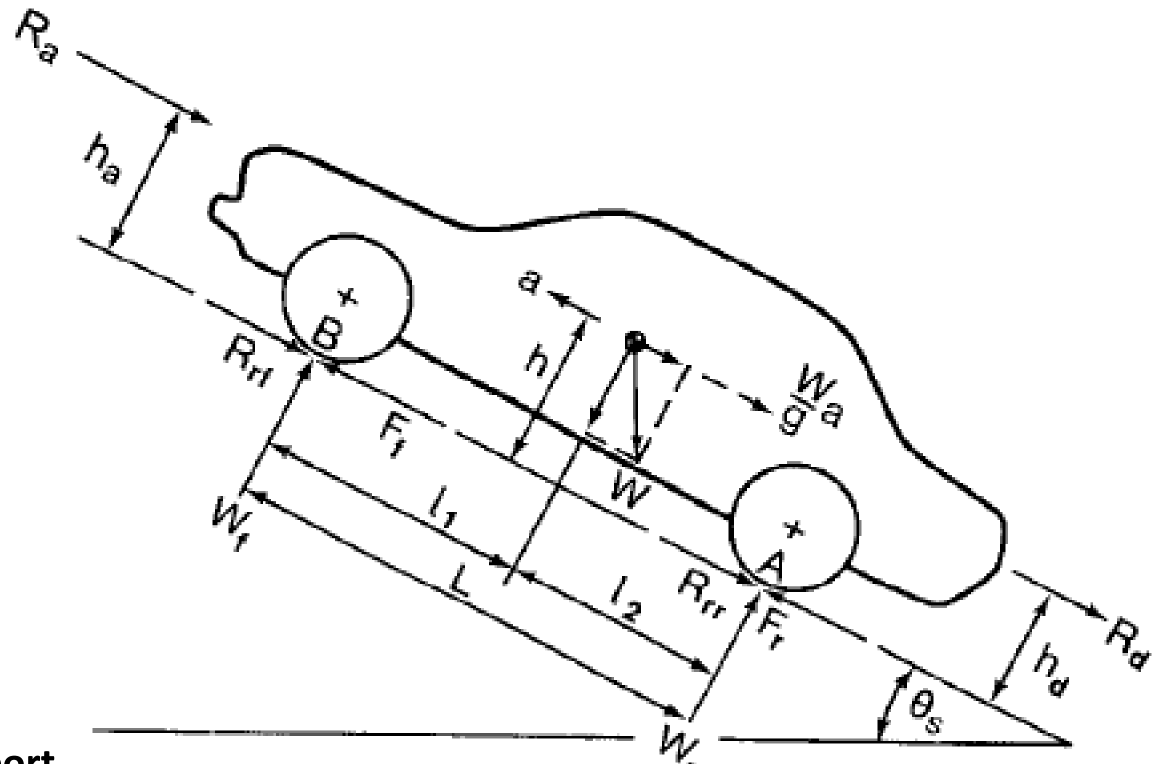
Acceleration performance

$$m\frac{d^2x}{dt^2} = \frac{W}{g}a = F_f + F_r - R_a - R_{rf} - R_{rr} - R_d - R_g$$

$$F_f + F_r - \left(R_a + R_{rf} + R_{rr} + R_d + R_g + \frac{aW}{g}\right) = 0$$

When rearranged

$$F = R_a + R_r + R_d + R_g + \frac{aW}{g}$$



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To predict the maximum tractive effort that the tire-ground contact can support, the normal loads on the axles have to be determined

$$W_f = \frac{l_2}{L}W - \frac{h}{L}\left(R_a + \frac{aW}{g} + R_d \pm W\sin\theta_s\right)$$

$$W_r = \frac{l_1}{L}W + \frac{h}{L}\left(R_a + \frac{aW}{g} + R_d \pm W\sin\theta_s\right)$$

When the vehicle is climbing up a hill, the **negative** sign is used for the term $Whsin\theta_s$

$$\boxed{W_f = \frac{l_2}{L}W - \frac{h}{L}(F - R_r)}$$

$$F = R_a + R_r + R_d + R_g + \frac{aW}{g}$$

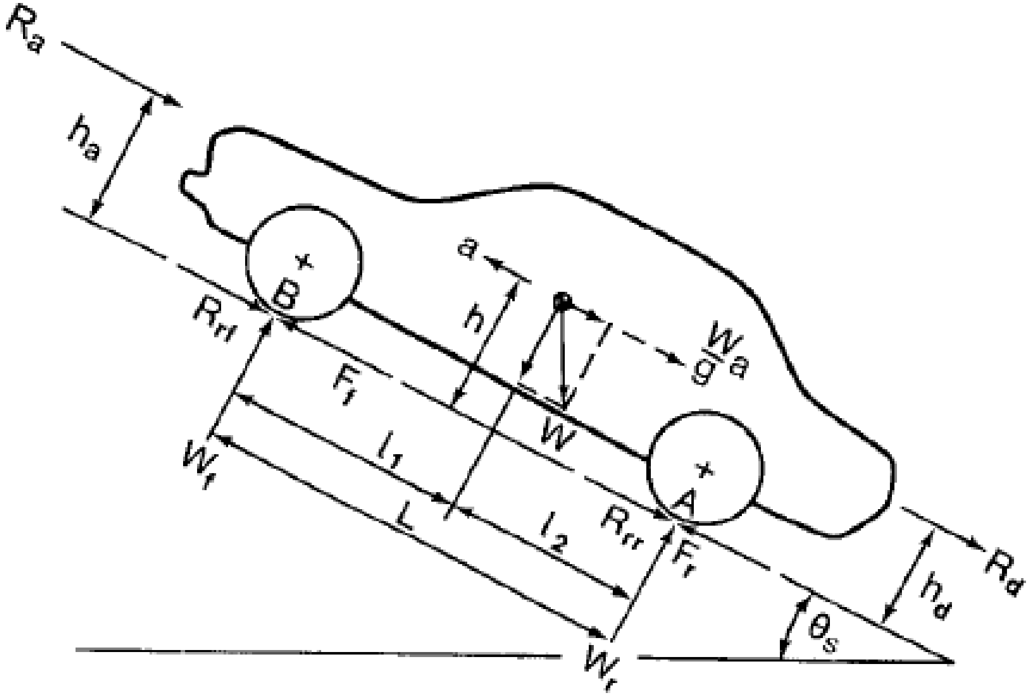
$$\boldsymbol{R_g = W \cdot sin\theta}$$

When the vehicle is climbing up a hill, the **positive** sign is used for the term $Whsin\theta_s$

$$\boxed{W_r = \frac{l_1}{L}W + \frac{h}{L}(F - R_r)}$$

Acceleration performance

- There is a maximum tractive effort that the tire-ground contact can support;
- There is a maximal acceleration of a vehicle proportional to the friction under its tires;
- Max tractive effort depends of tire road interaction – coefficient of road adhesion;
- ***Rolling resistance*** is dependent upon the rolling resistance coefficient and the weight of the vehicle $\boldsymbol{R_r = f_r \cdot W}$;



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$$F_{\max} = \mu W_f = \mu \left[\frac{l_2}{L} W - \frac{h}{L} (F_{\max} - R_r) \right]$$

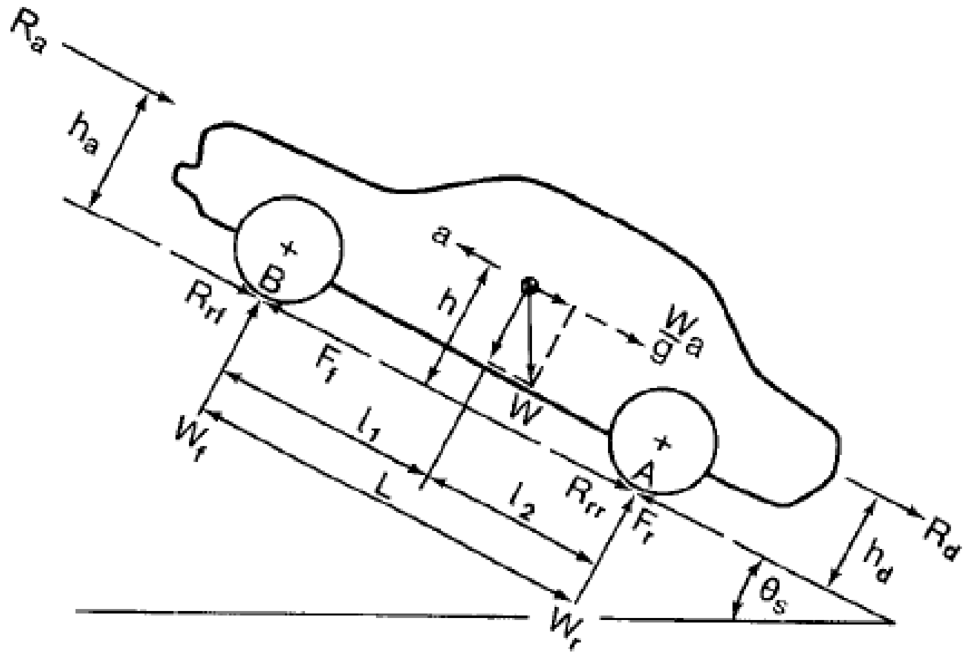
$$F_{\max} = \mu W_r = \mu \left[\frac{l_1}{L} W + \frac{h}{L} (F_{\max} - R_r) \right]$$

$$F_{\max} = \frac{\mu W (l_2 + f_r h)/L}{1 + \mu h/L}$$

$$F_{\max} = \frac{\mu W (l_1 - f_r h)/L}{1 - \mu h/L}$$

Gradability

- Gradability is usually defined as the maximum grade vehicle can negotiate at a given steady speed.
- Slope at a constant speed, the tractive effort has overcome grade resistance, rolling resistance, ar aerodynamic resistance



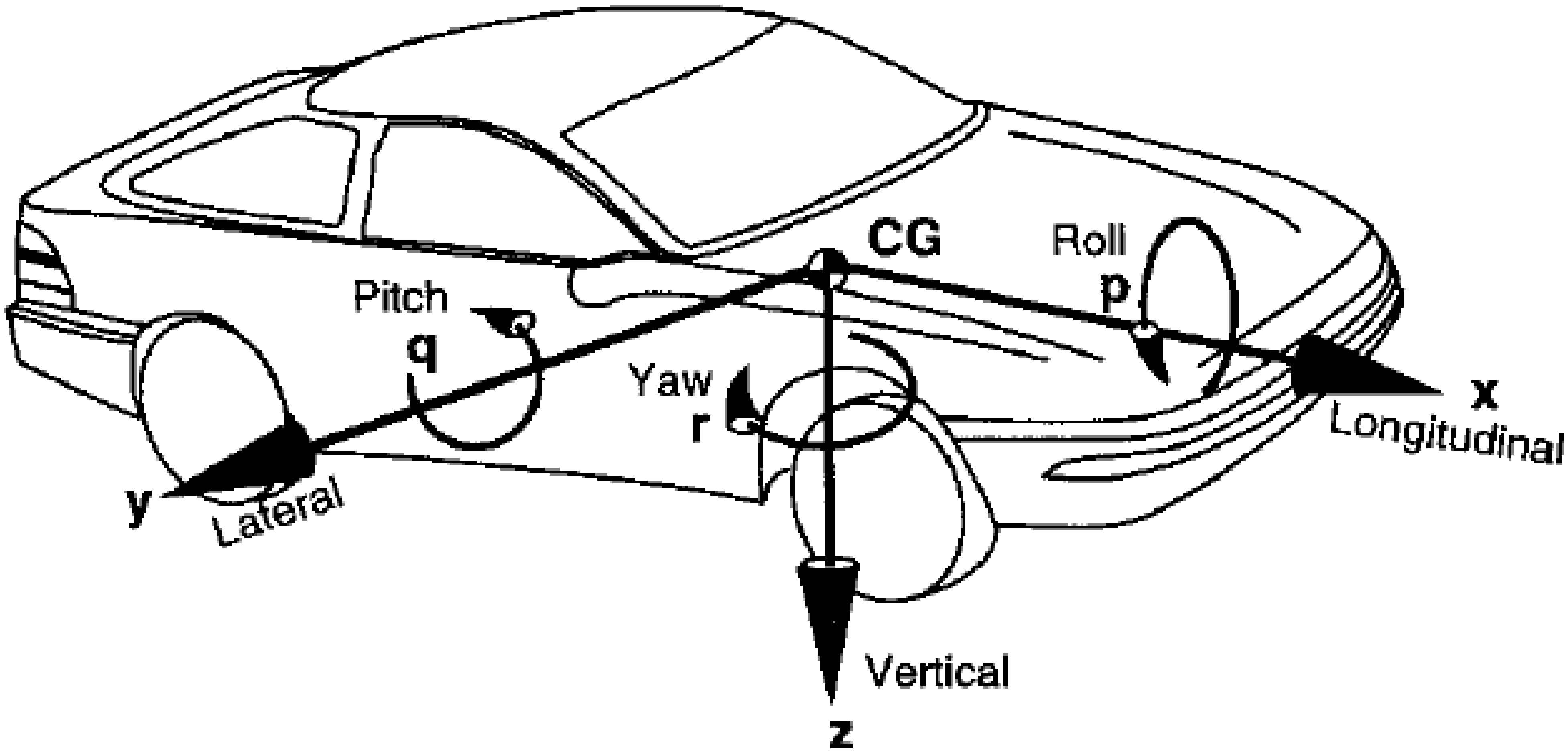
WONG, J. Y. 2001. *Theory of Ground Vehicles*, Wiley.

$$F = W \sin \theta_s + R_r + R_a$$

- For small angles $\boldsymbol{sin\theta_s = tan\theta_s}$ so the grade resistance can be replaced by grad in %
- **The limits of tractive effort set by the nature of tire-road adhesion usually determine the maximum gradability of the vehicle**

$$G = \frac{1}{W}(F - R_r - R_a) = \frac{F_{\text{net}}}{W}$$

Vehicle dynamics
lateral motion



GILLESPIE, T. D. 1992. *Fundamentals of Vehicle Dynamics*, Society of Automotive Engineers.

Lateral force;
Cornering

$$\cot \delta_o = (B/2 + e_2)/e_1$$

$$\cot \delta_i = (B/2 - e_2)/e_1$$

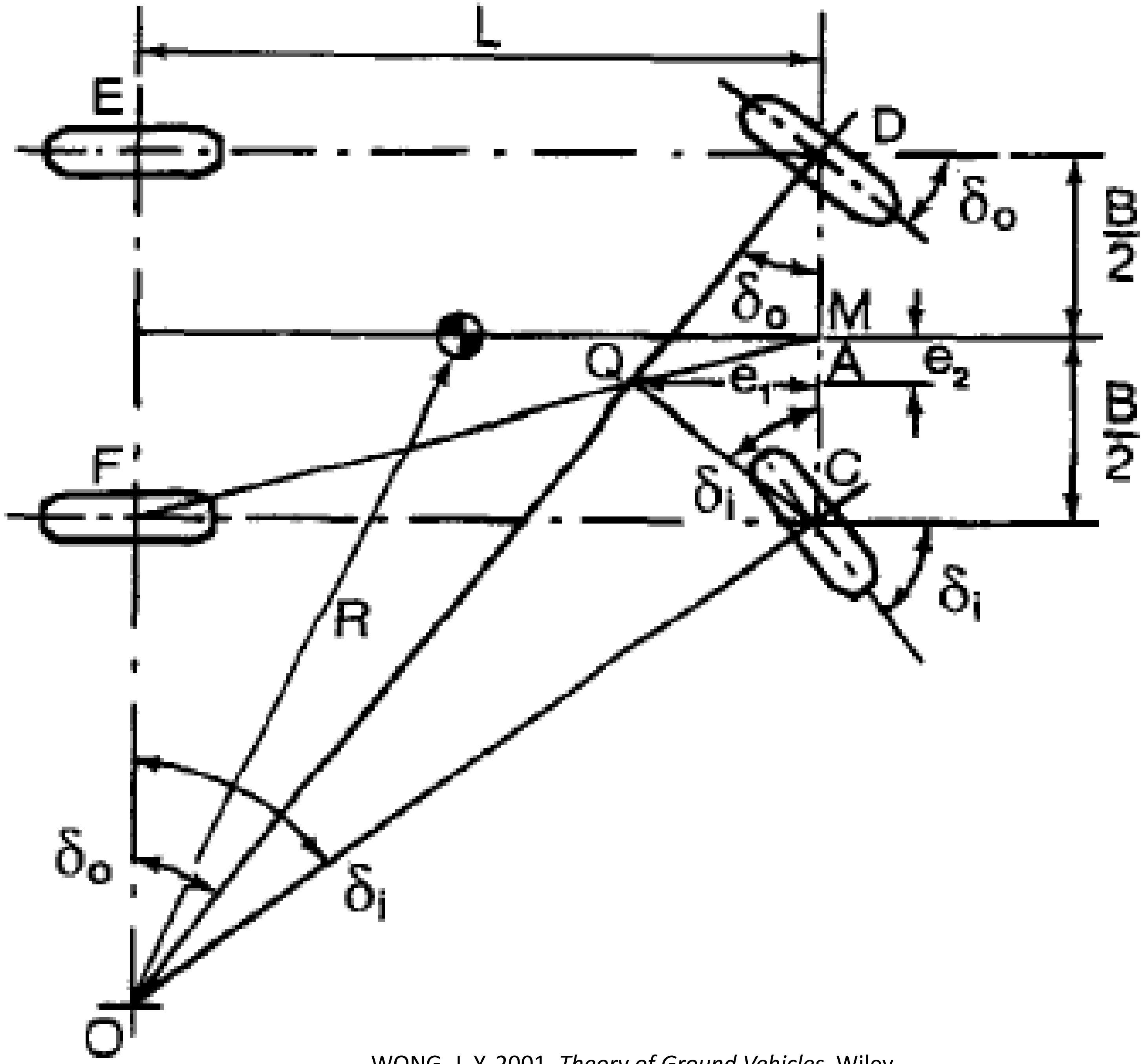
$$\cot \delta_o - \cot \delta_i = 2e_2/e_1$$

Since triangle MAQ is similar to triangle MCF,

$$\frac{e_2}{e_1} = \frac{B/2}{L}$$

Ackerman condition

$$\boxed{\cot \delta_o - \cot \delta_i = B/L}$$



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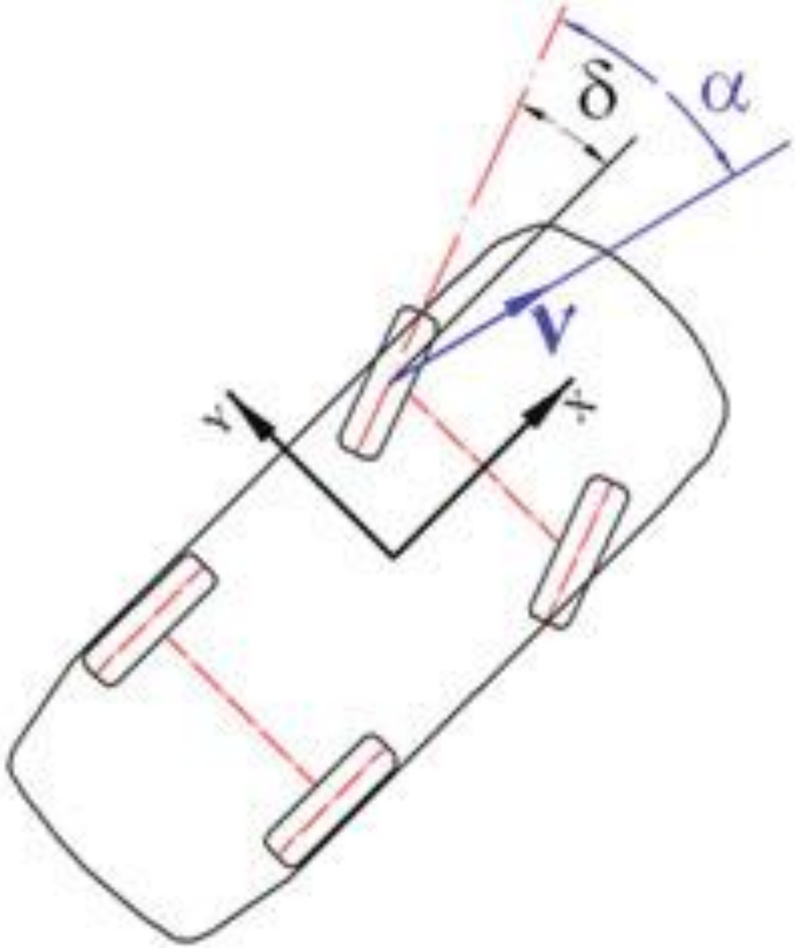
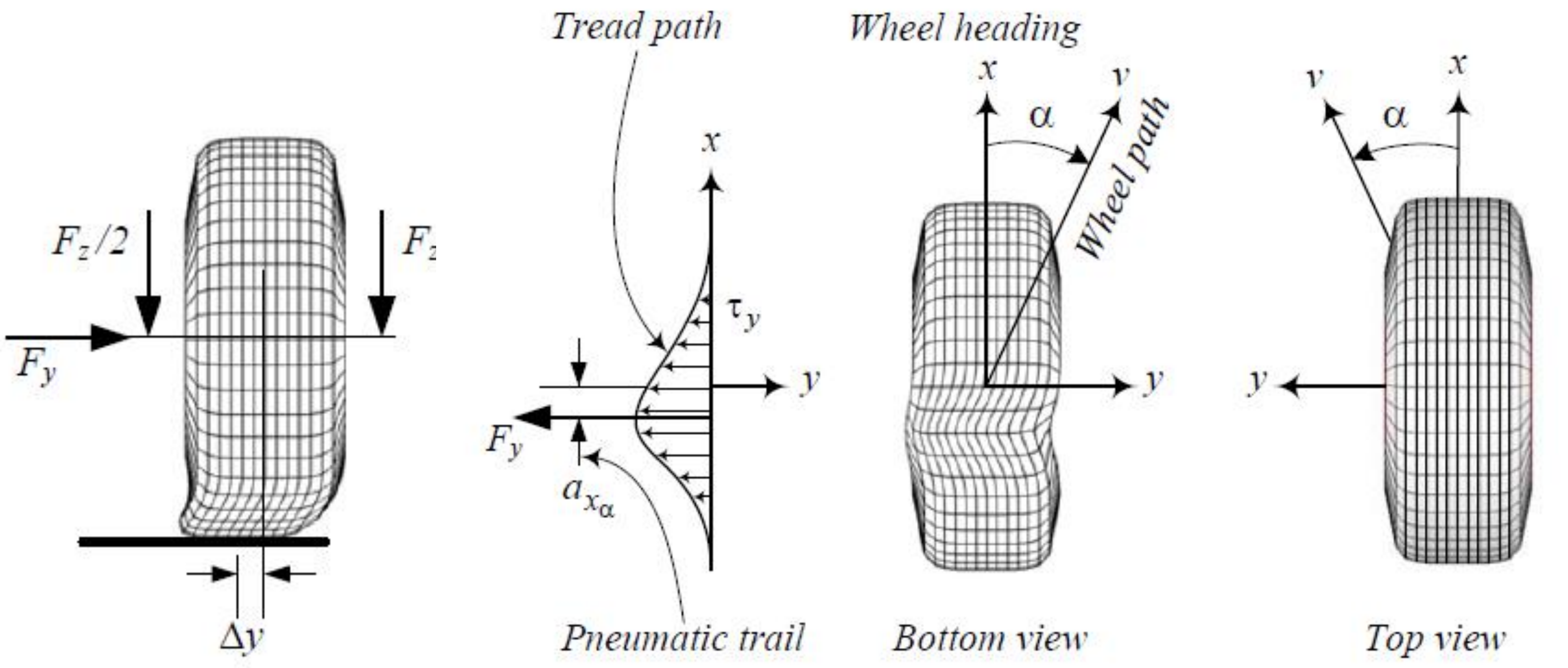
Lateral force;
Side slip angle



HEIßING, B. & ERSOY, M. 2010. *Chassis Handbook: Fundamentals, Driving Dynamics, Components, Mechatronics, Perspectives*, Vieweg+Teubner Verlag.



JAZAR, R. N. 2008. *Vehicle Dynamics: Theory and Application*, Springer US.

To provide a measure for comparing the cornering behavior of different tires, a parameter called cornering stiffness $\boldsymbol{C_\alpha}$ is established **. It is defined as the derivative of the cornering force $\boldsymbol{F_y}$ with respect to slip angle $\boldsymbol{\alpha}$ evaluated at zero slip angle**

$$C_\alpha = \left.\frac{\partial F_{y\alpha}}{\partial \alpha}\right|_{\alpha=0}$$

Lateral force;

Cornering

- The handling characteristics of the vehicle depend, to a great extent, on the relationship between the slip angles of the front and rear tires, $\boldsymbol{\alpha_f}$ and $\boldsymbol{\alpha_r}$
- The steer angle $\boldsymbol{\delta_f}$ required to negotiate a given curve is a function of not only the turning radius $\boldsymbol{R}$, but also the front and rear slip angles $\boldsymbol{\alpha_f}$ and $\boldsymbol{\alpha_r}$

$$\delta_f - \alpha_f + \alpha_r = L/R$$

- The slip angles are dependent on the side forces acting on the tires and their cornering stiffness. The cornering forces on the front and rear tires can be determined from the dynamic equilibrium of the vehicle in the lateral direction.

$$F_{yf} = \frac{W}{g}\frac{V^2}{R}\frac{l_2}{L}$$

$$F_{yr} = \frac{W}{g}\frac{V^2}{R}\frac{l_1}{L}$$

The normal load on each of the front tires

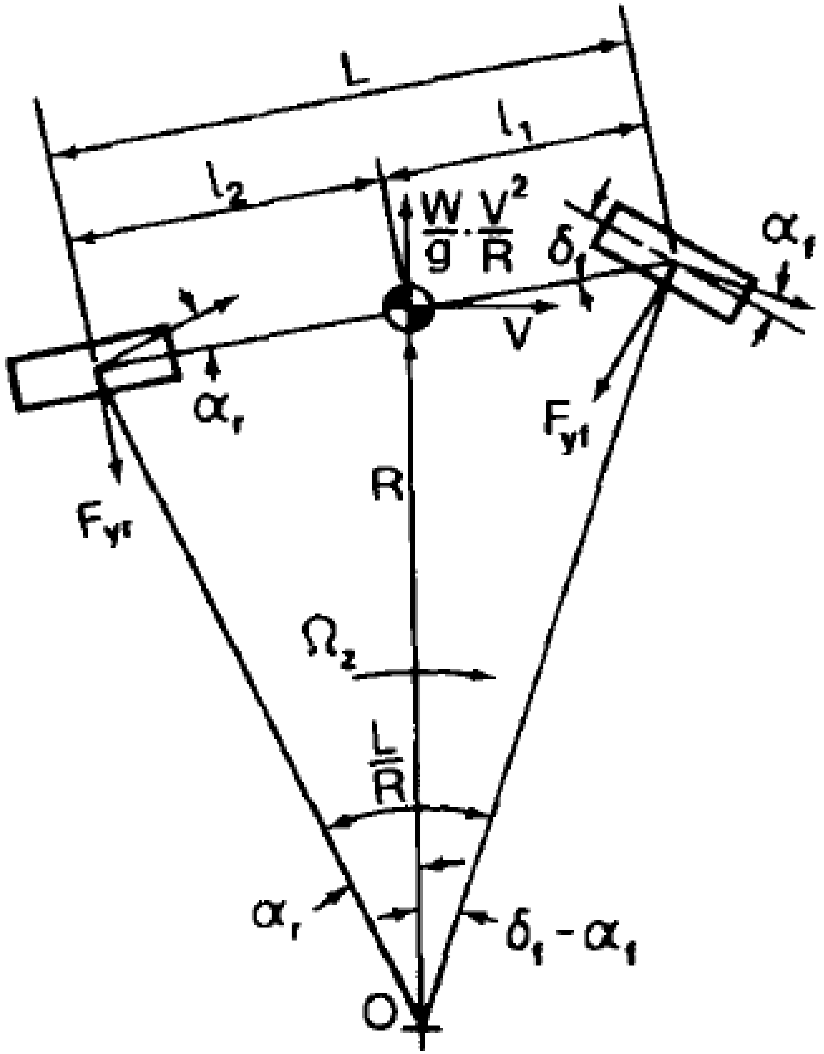
$$W_f = Wl_2/2L$$

$$W_r = Wl_1/2L$$

Hence

$$F_{yf} = 2W_f\frac{V^2}{gR}$$

$$F_{yr} = 2W_r\frac{V^2}{gR}$$



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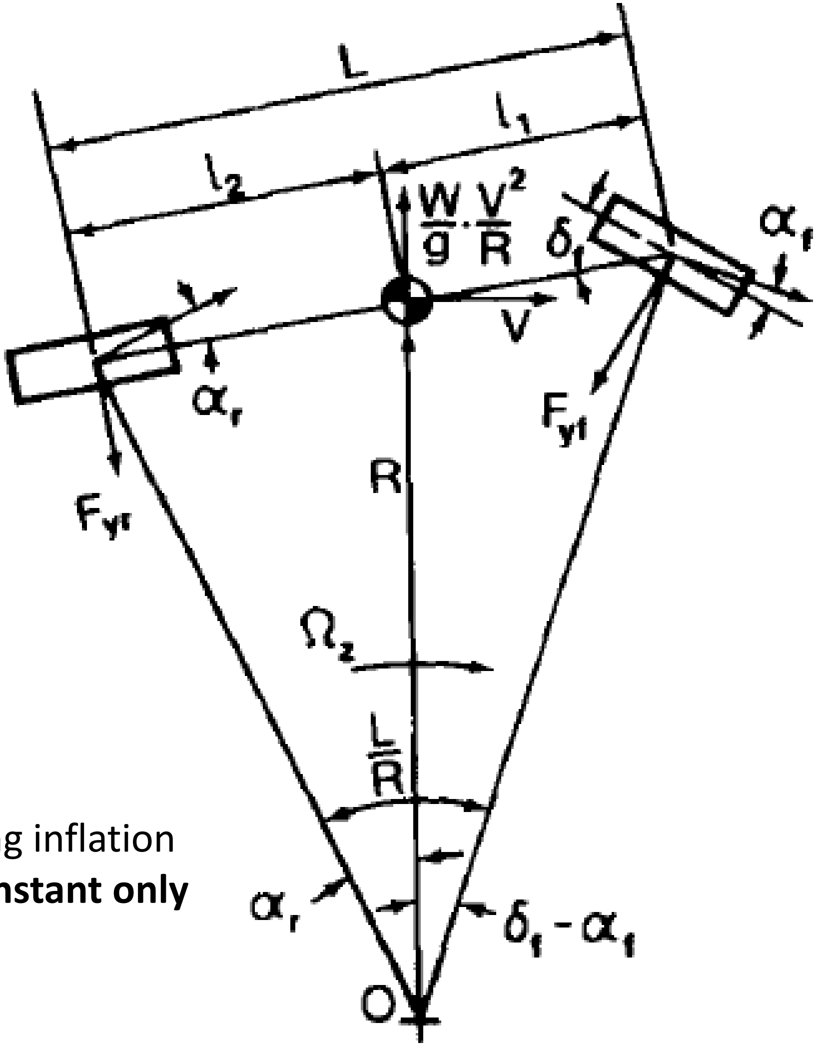
Lateral force;

Cornering

Within a certain range, the slip angle and cornering force may be considered to be linearly related with a constant cornering stiffness,

$$C_\alpha = \left.\frac{\partial F_{y\alpha}}{\partial \alpha}\right|_{\alpha=0} \quad \Longrightarrow \quad \alpha_f = \frac{F_{yf}}{2C_{\alpha f}} = \frac{W_f}{C_{\alpha f}}\frac{V^2}{gR}, \qquad \alpha_r = \frac{F_{yr}}{2C_{\alpha r}} = \frac{W_r}{C_{\alpha r}}\frac{V^2}{gR}$$

For simplification of this example it was assumed that the vehicle has two tires with double stiffness



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The cornering stiffness of a given tire varies with a number of operational parameters, including inflation pressure, normal load, tractive (or braking) effort, and lateral force. **It may be regarded as a constant only within a limited range of operating conditions**

- The steer angle is

$$\delta_f - \alpha_f + \alpha_r = L/R$$

- Hence

$$\delta_f = \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}\right)\frac{V^2}{gR} = \frac{L}{R} + K_{us}\frac{V^2}{gR} = \frac{L}{R} + K_{us}\frac{a_y}{g}$$

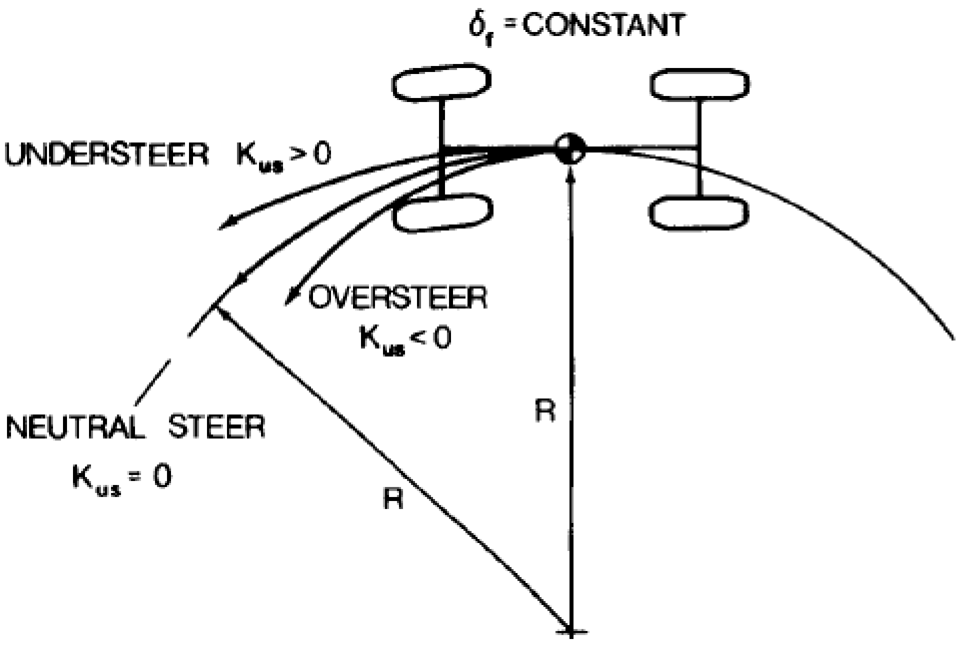
- $\boldsymbol{K_{us}}$ the understeer coefficient and is expressed in radians. Function of weight distribution and tire cornering stiffness;
- Dependent on the values of the understeer coefficient $\boldsymbol{K_{us}}$ or the relationship between the slip angles of the front and rear tires, the steady-state handling characteristics may be classified into three categories: **neutral steer, understeer, and oversteer**

Lateral force;

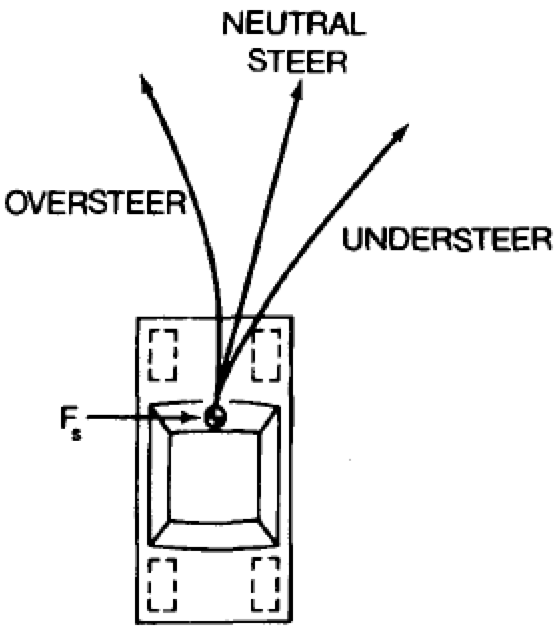
Cornering; Neutral steer

- $\boldsymbol{K_{us}} = \mathbf{0}$ equivalent to the slip angles of the front and rear tires being equal $\alpha_f = \alpha_r,\ and\ \frac{W_f}{C_{\alpha f}} = \frac{W_r}{C_{\alpha r}}$
- In such case the angle required to negotiate a given curve is independent of forward speed and is given by

$$\delta_f = L/R$$



- When a neutral steer vehicle is accelerated in a constant radius turn, the driver should maintain the same steering wheel position.
- When a neutral steer vehicle originally moving along a straight line is subjected to a side force acting at the center of gravity, equal slip angles will be developed at the front and rear tires. As a result, the vehicle follows a straight path at an angle to the original.

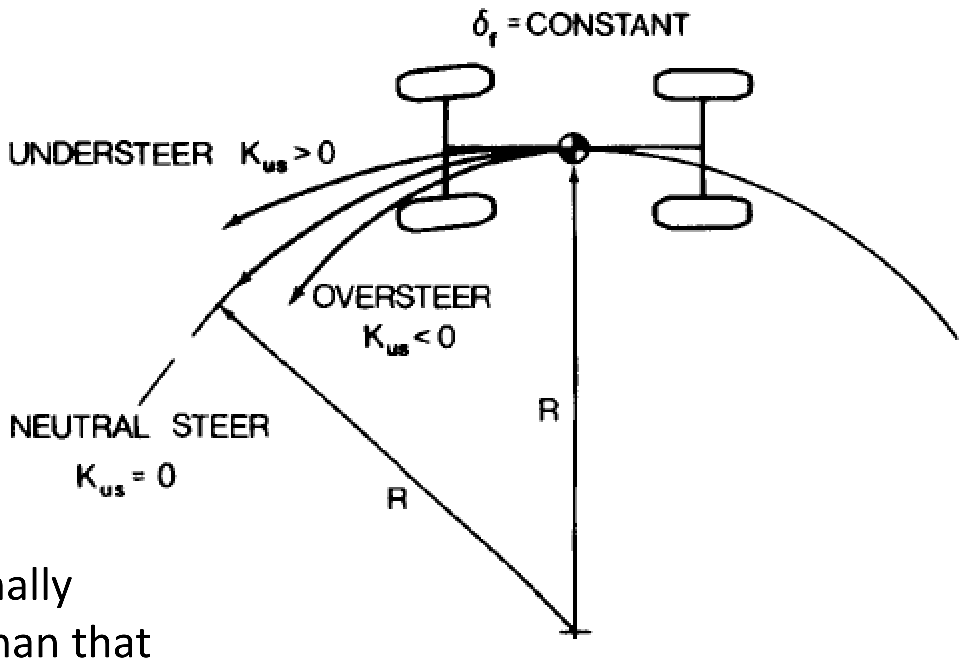


WONG, J. Y. 2001. *Theory of Ground Vehicles*, Wiley.

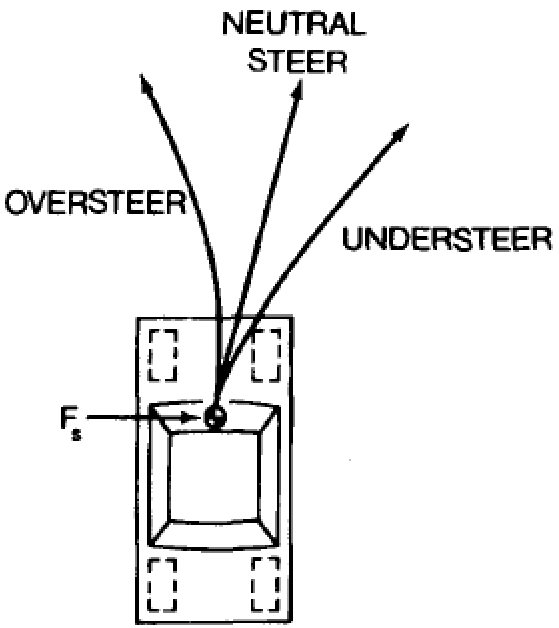
Lateral force;

Cornering; Understeer

- $\boldsymbol{K_{us} > 0}$ which is equivalent to the slip angles of the front and rear tires being equal $\alpha_f > \alpha_r,\ and\ \frac{W_f}{C_{\alpha f}} > \frac{W_r}{C_{\alpha r}}$
- In such case the steer angle required to negotiate a given **curve increases with the square of vehicle forward speed** (or lateral acceleration).



- When a side force acts at the center of gravity of an understeer vehicle originally moving along a straight line, the front tires will develop a slip angle greater than that of the rear tires
- As a result, a yaw motion is initiated, and the vehicle turns away from the side force;
- At the same steering wheel position and vehicle forward speed, the turning radius of an understeer vehicle is larger than that of a neutral steer vehicle;
- There is a characteristic speed for and understeer vehicles. It is the speed at which the steer angle required to negotiate a turn is equal to $2L/R$



$$V_{char} = \sqrt{\frac{gL}{K_{us}}}$$

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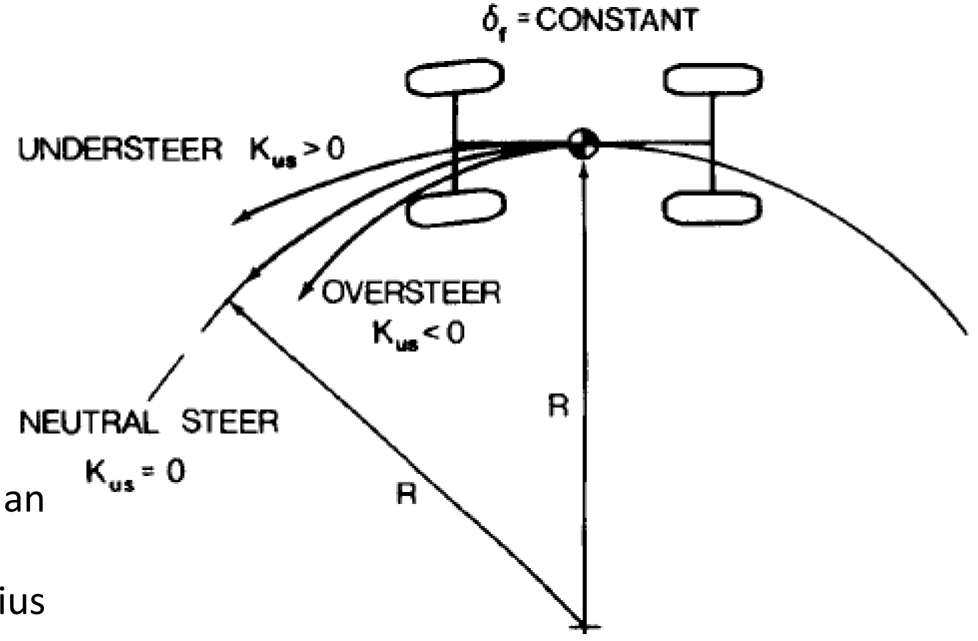
Lateral force;

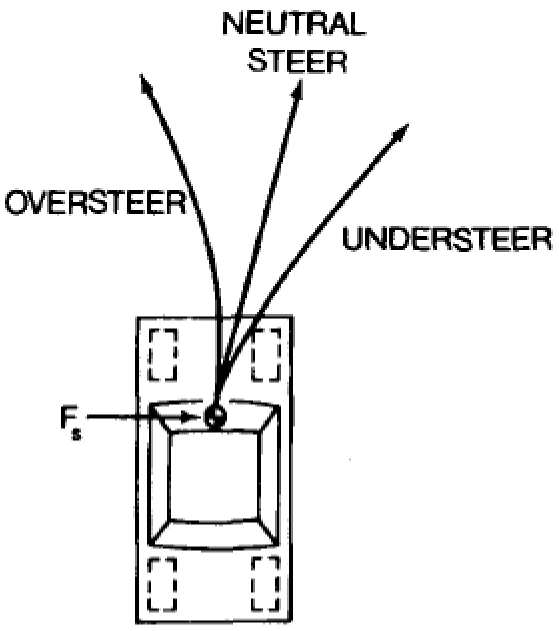
Cornering; Oversteer

- $\boldsymbol{K_{us} < 0}$ which is equivalent to the slip angles of the front and rear tires being equal $\alpha_f < \alpha_r,\ and\ \frac{W_f}{C_{\alpha f}} < \frac{W_r}{C_{\alpha r}}$



- In such case the steer angle required to negotiate a given curve **decreases** with an **increase** of vehicle forward speed (or lateral acceleration)
- when a vehicle is accelerated with the steering wheel fixed, the turning radius decreases,
- For the same steering wheel position and vehicle the turning radius of an oversteer vehicle is smaller than that of a neutral steer vehicle.
- When a side force acts at the center of gravity of an oversteer vehicle originally moving along a straight line, the front tires will develop a slip angle less than that of the rear tires
- As a result, a yaw motion is initiated, and the vehicle turns into the side force;
- For an oversteer vehicle, a critical speed $\boldsymbol{V_{crit}}$ can be identified. **It is the speed at which the steer angle required to negotiate any turn is zero, as shown.** The critical speed also represents the speed above which an oversteer vehicle exhibits directional instability.

$$V_{\text{crit}} = \sqrt{\frac{gL}{-K_{us}}}$$



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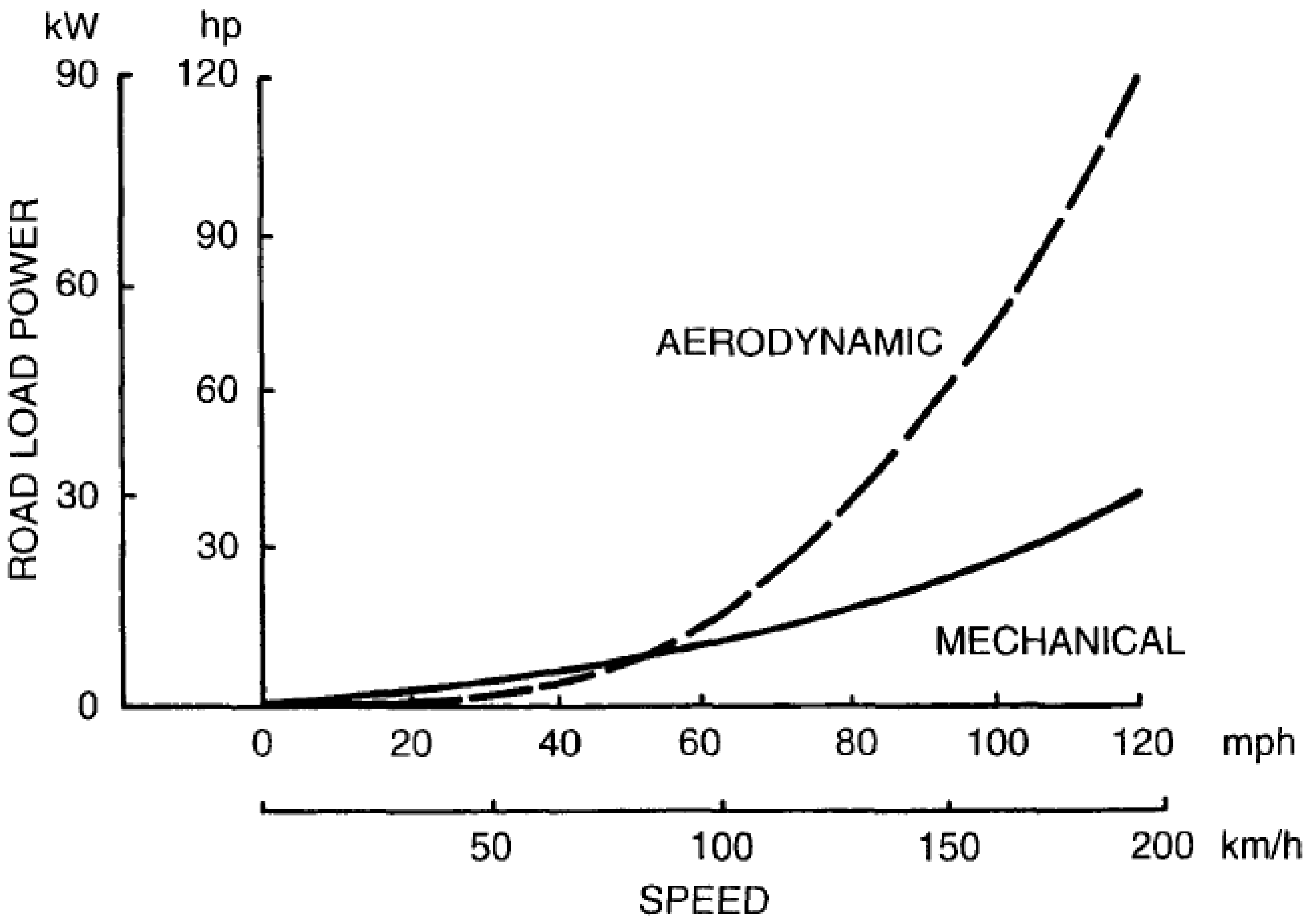
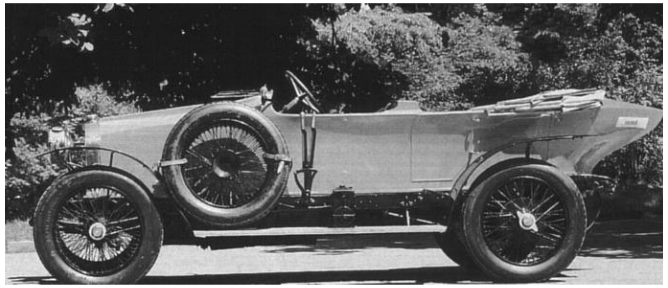
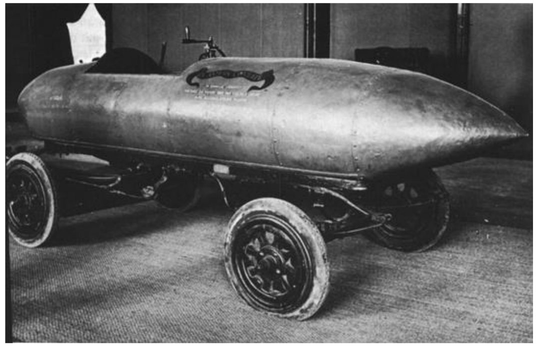
Road loads

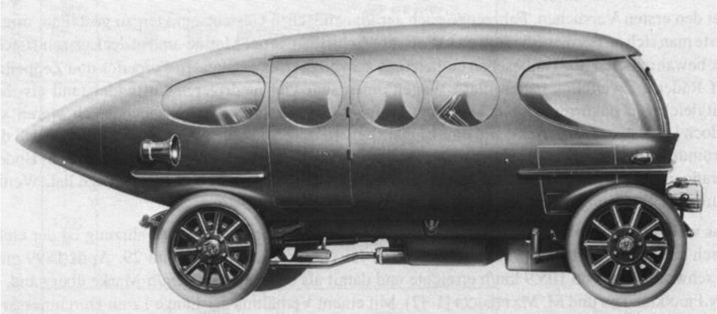


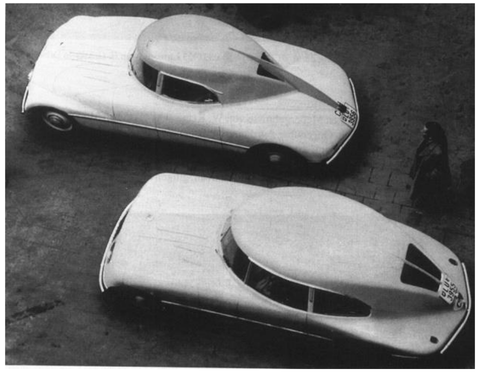
WONG, J. Y. 2001. *Theory of Ground Vehicles*, Wiley.

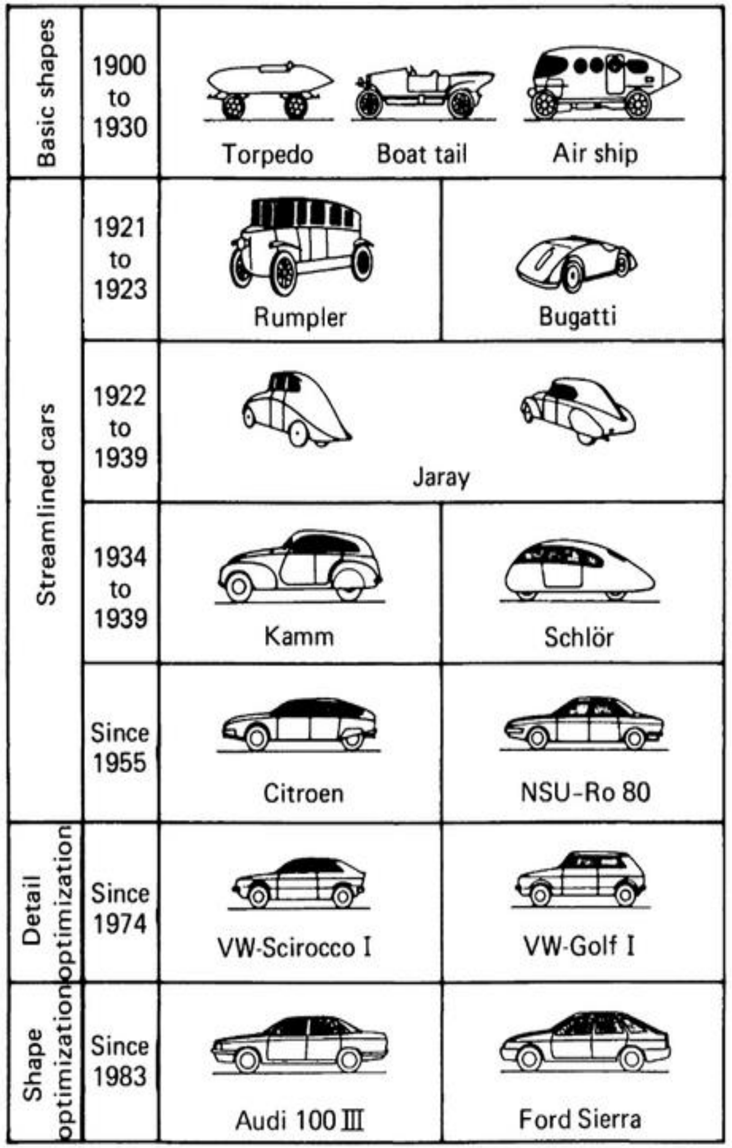
Aerodynamic resistance



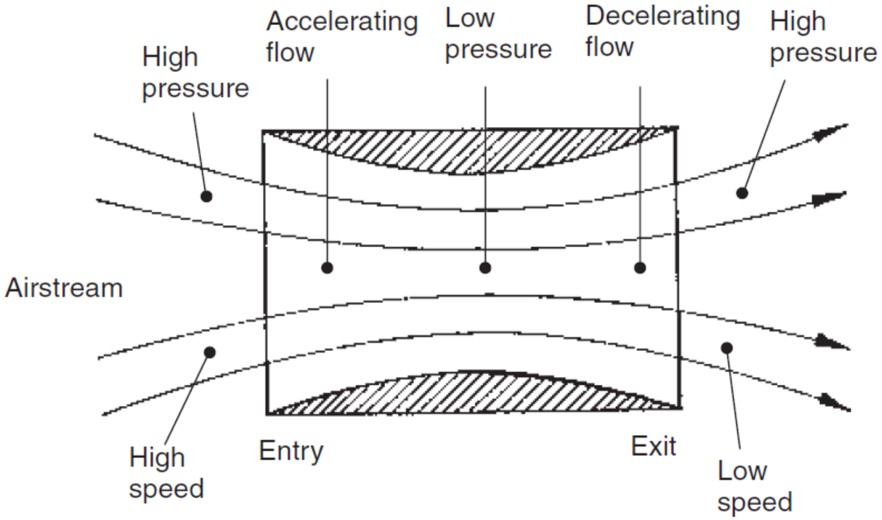


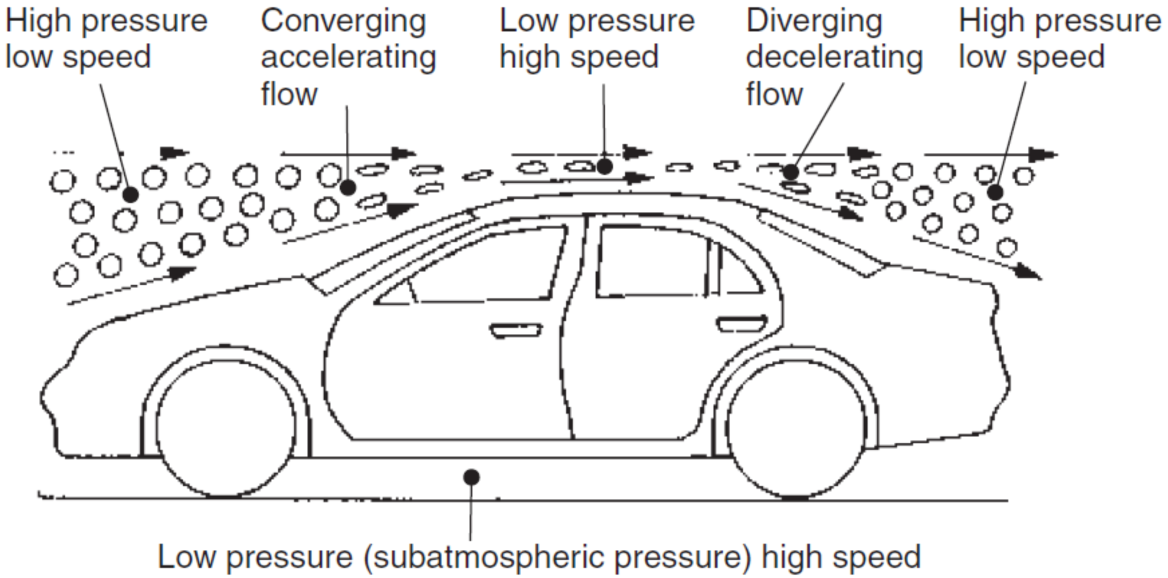


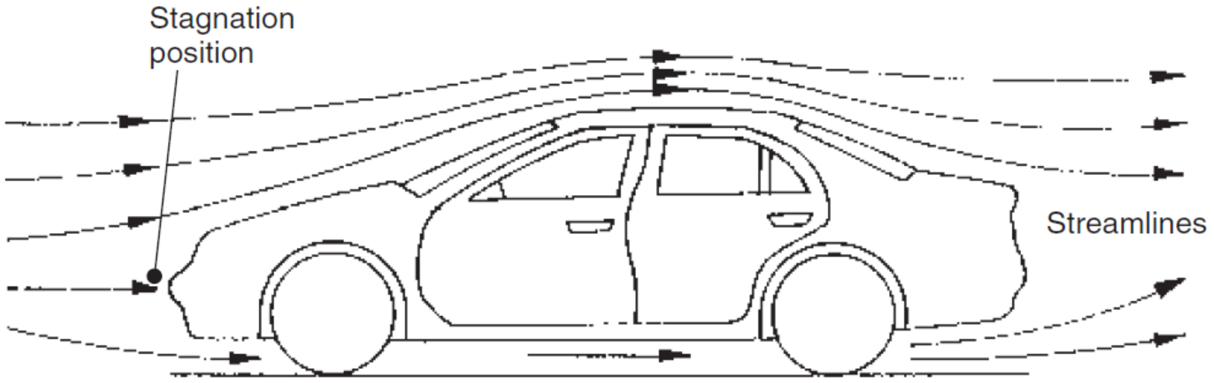




Flow mechanics







HEISLER, H. 2002. *Advanced Vehicle Technology*, Butterworth-Heinemann

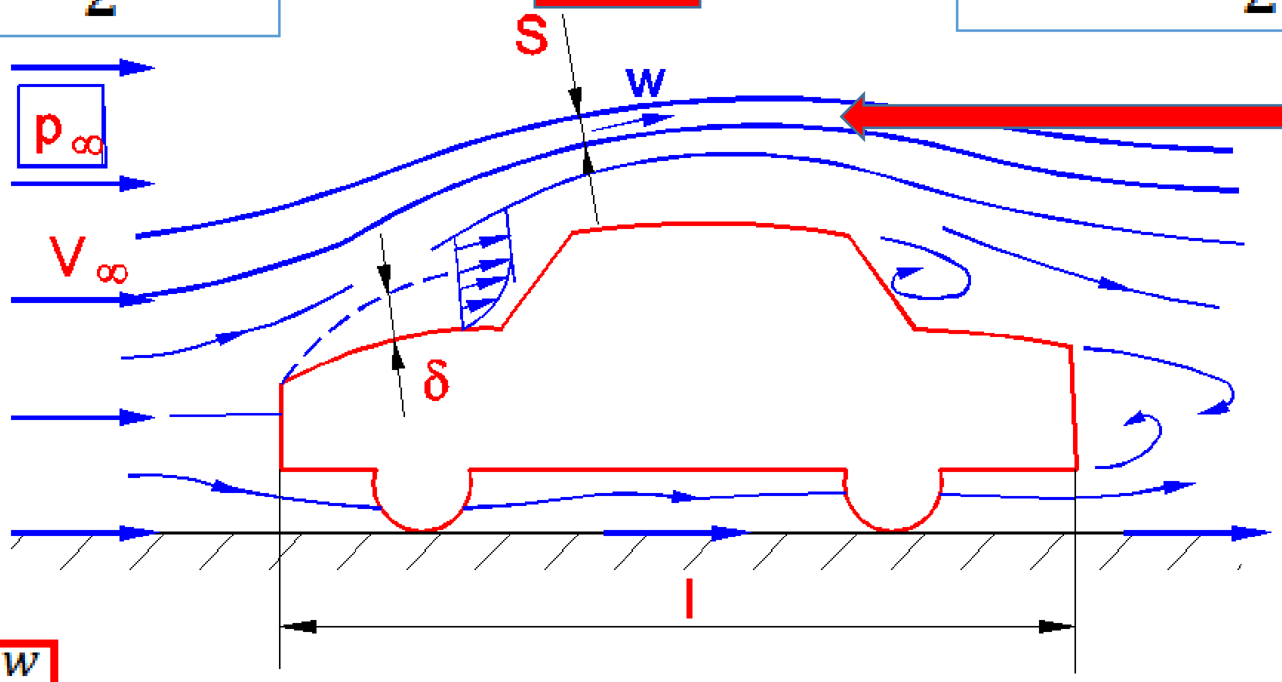
Flow mechanics

$$p + \frac{1}{2}\rho V^2 = const$$

$$p_{static} + p_{dynamic} = p_{total}$$

$$p_t = p_\infty + \frac{1}{2}\rho V_\infty{}^2$$

$$p_t = p + \frac{1}{2}\rho w^2$$



$$\boxed{C_p = \frac{p - p_\infty}{\frac{\rho V_\infty^2}{2}} = 1 - \frac{w}{V_\infty}}$$

If **V**=0, **Cp** = 1 (max value: stagnation)

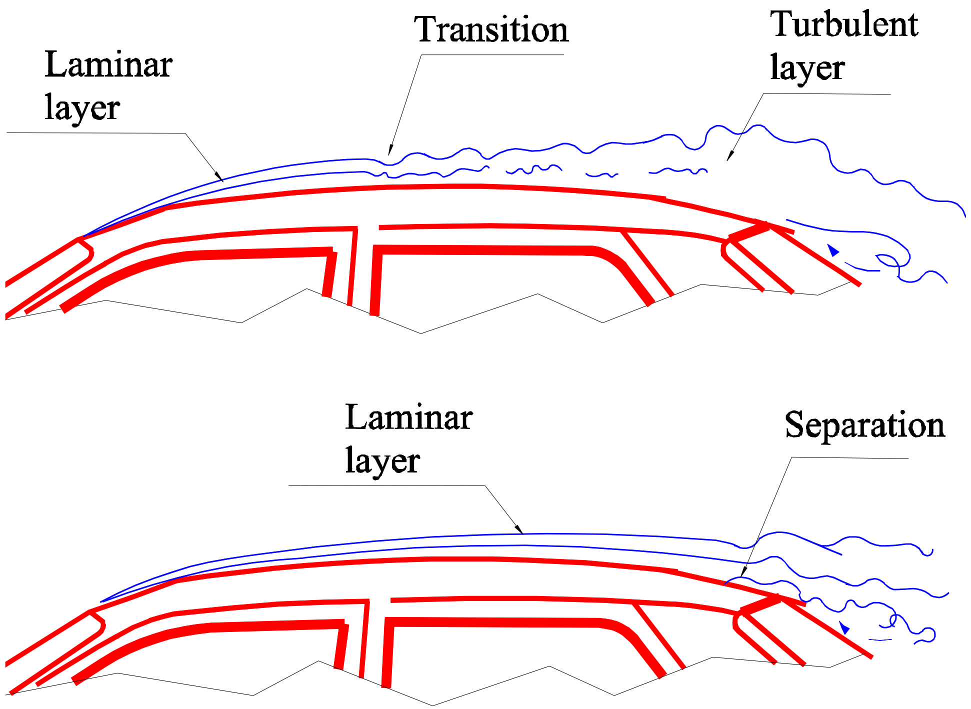
If **Cp** ↑ then **V** is ↓ & vice versa

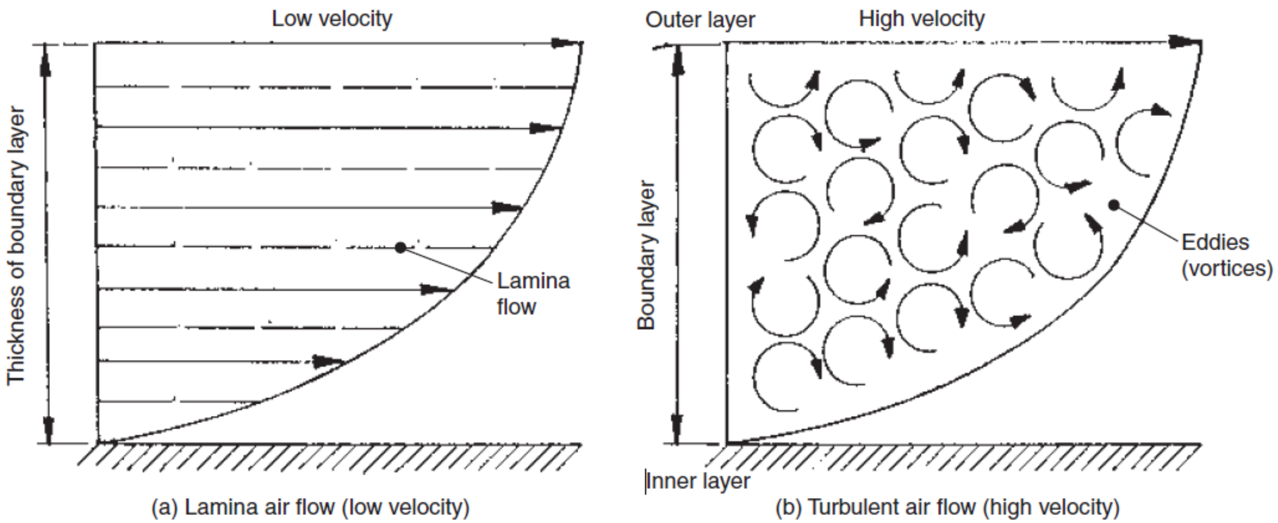
Separation of boundary layer

*The separation generates a type of drag called **pressure drag,** therefore, large emphasis is given to ensure attached flow as long as possible*

Turbulent BL separates later

due to intensive momentum transport from outside of boundary layer. Because of long attachment this stage generates **greater frictional drag**





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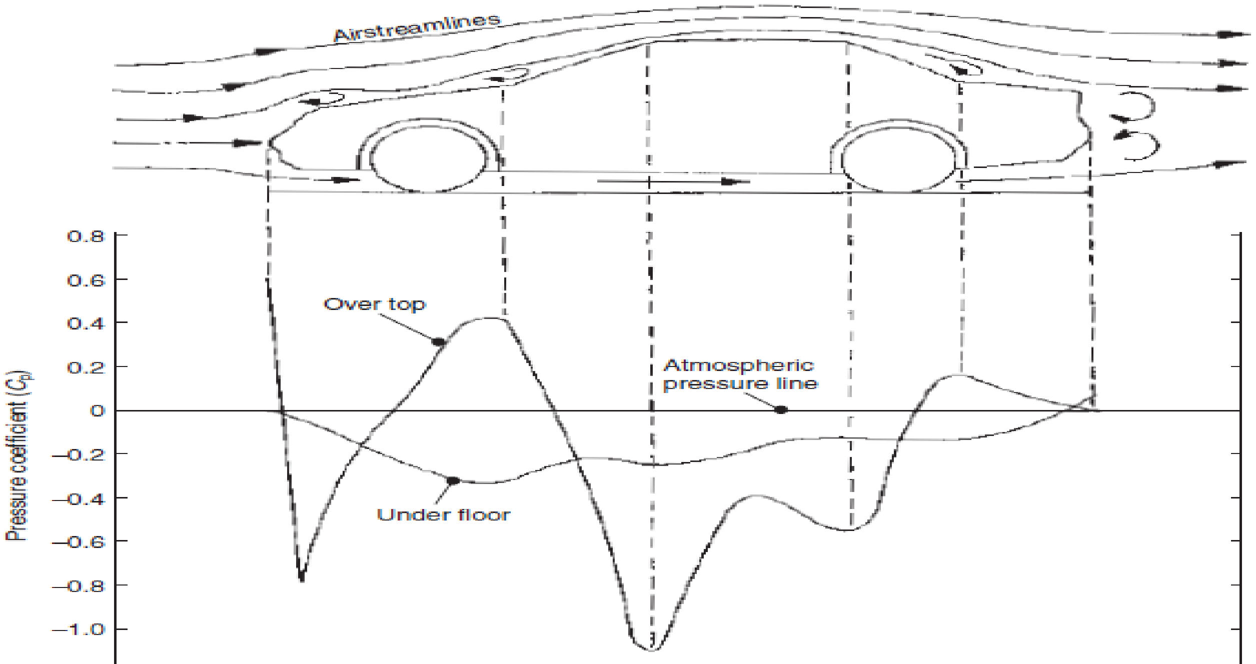
Pressure coefficient

(Usually presented graphically on the vehicle outline)



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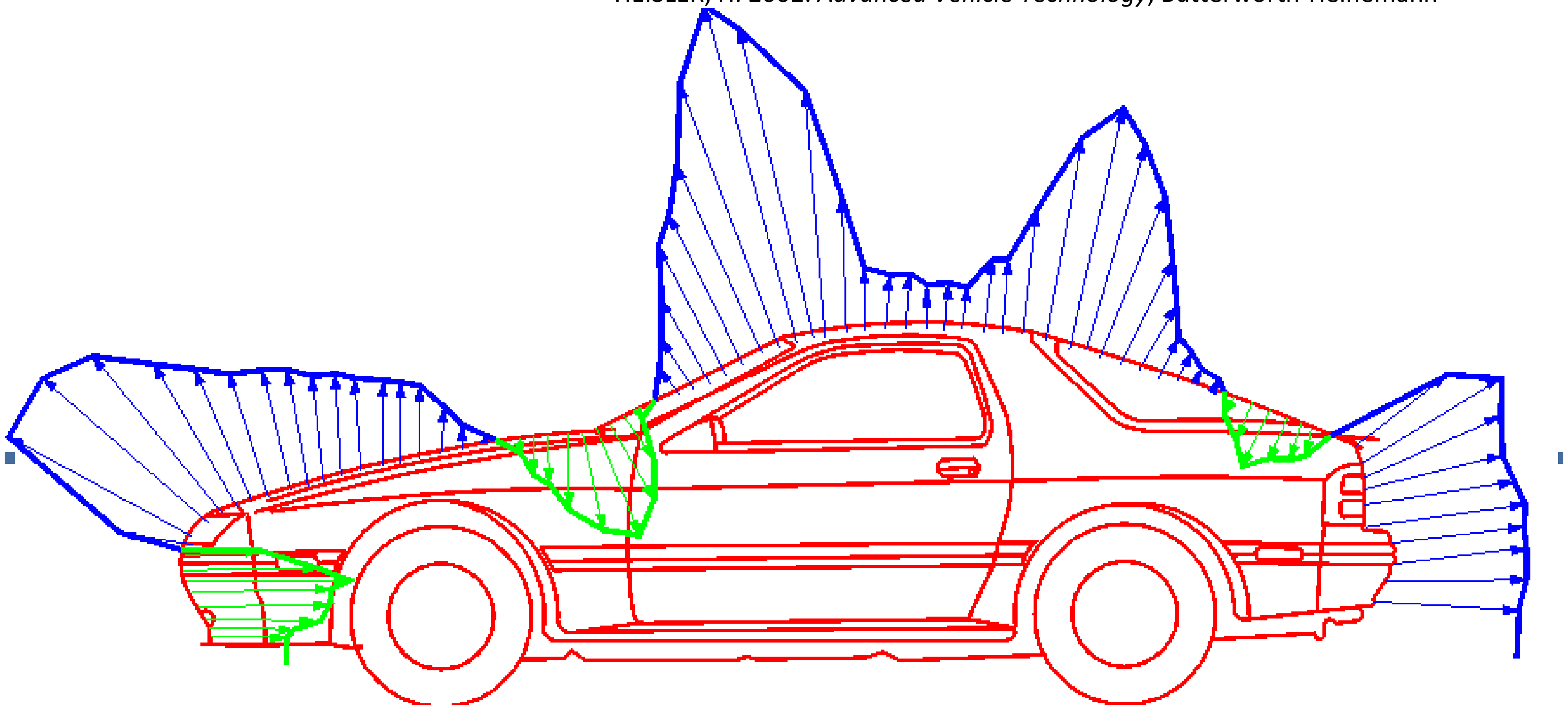
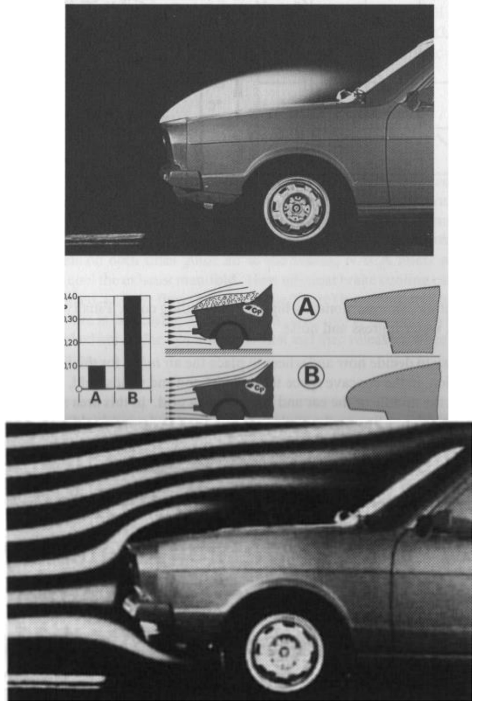
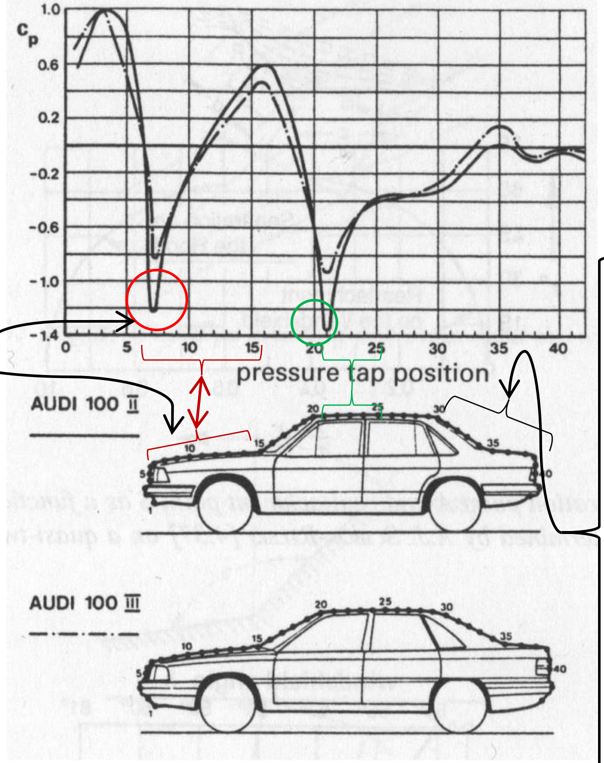
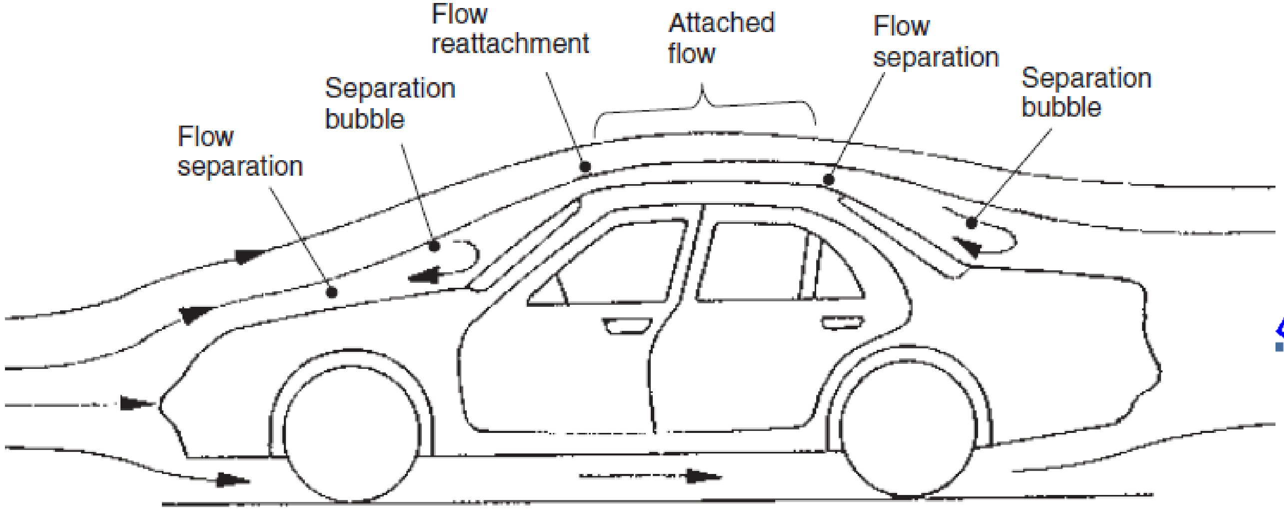


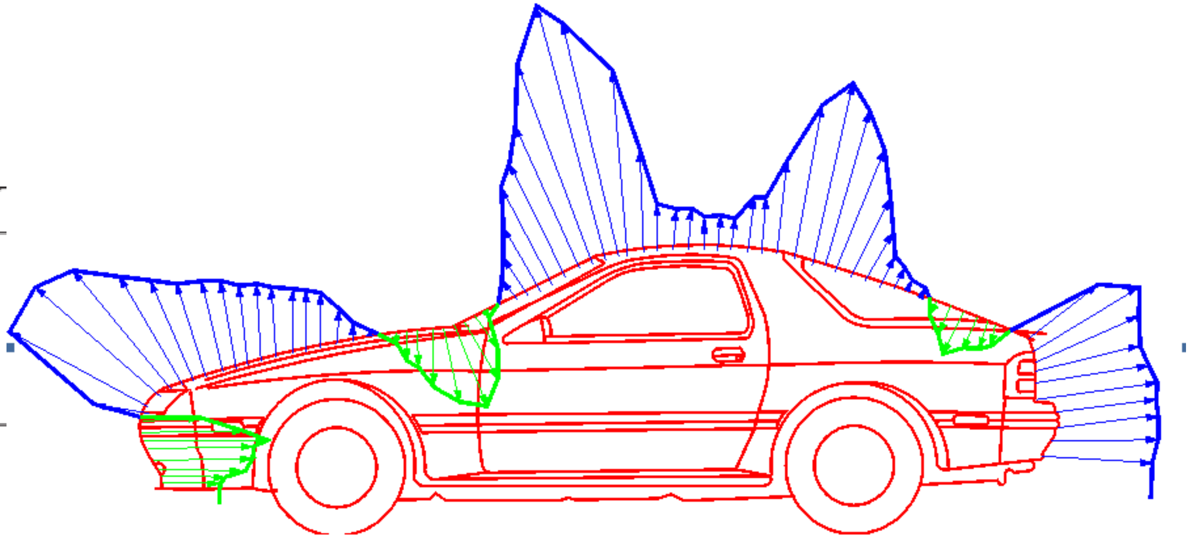




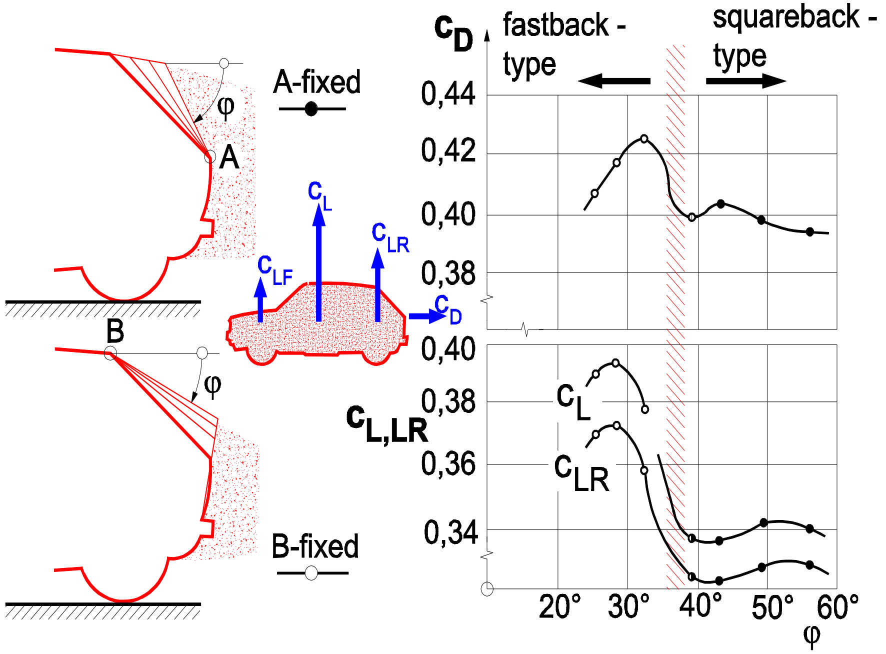
HUCHO, W. H. 2013. *Aerodynamics of Road Vehicles: From Fluid Mechanics to Vehicle Engineering*, Elsevier Science.

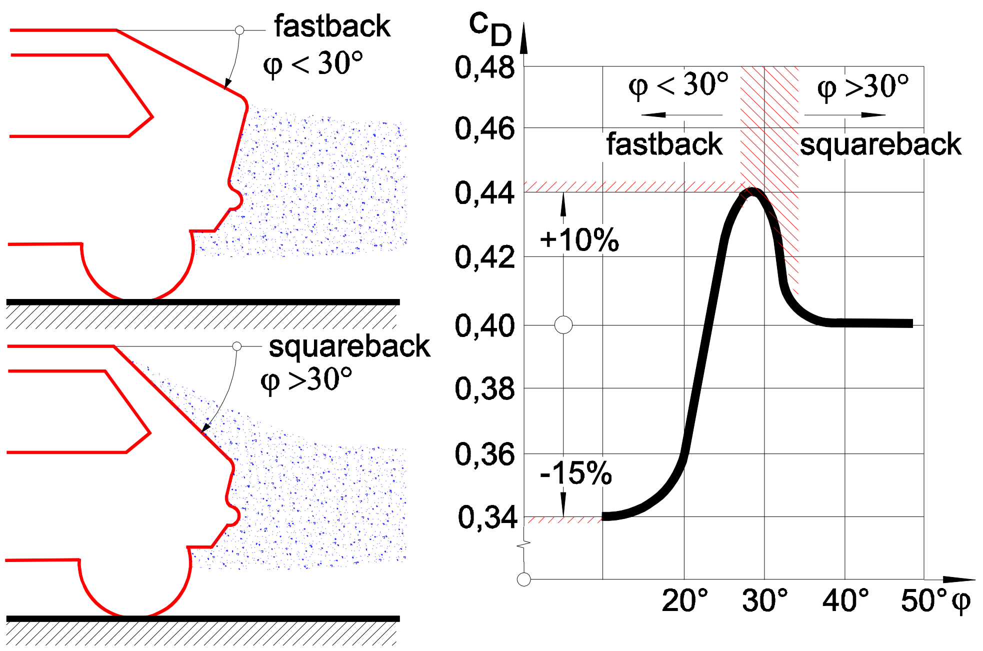


HEISLER, H. 2002. *Advanced Vehicle Technology*, Butterworth-Heinemann.



Rear of a car
(influence of the design on the aerodynamics performance)





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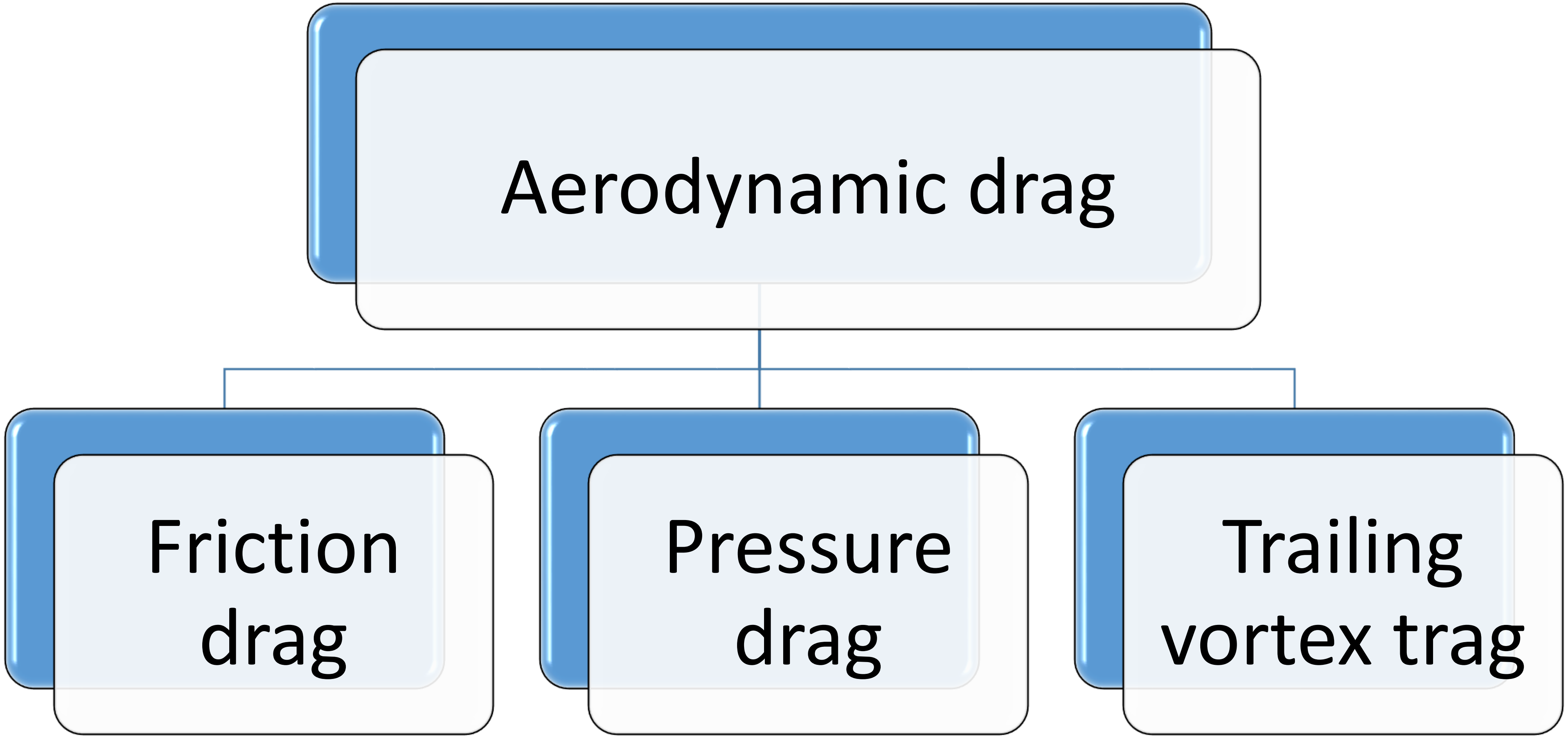
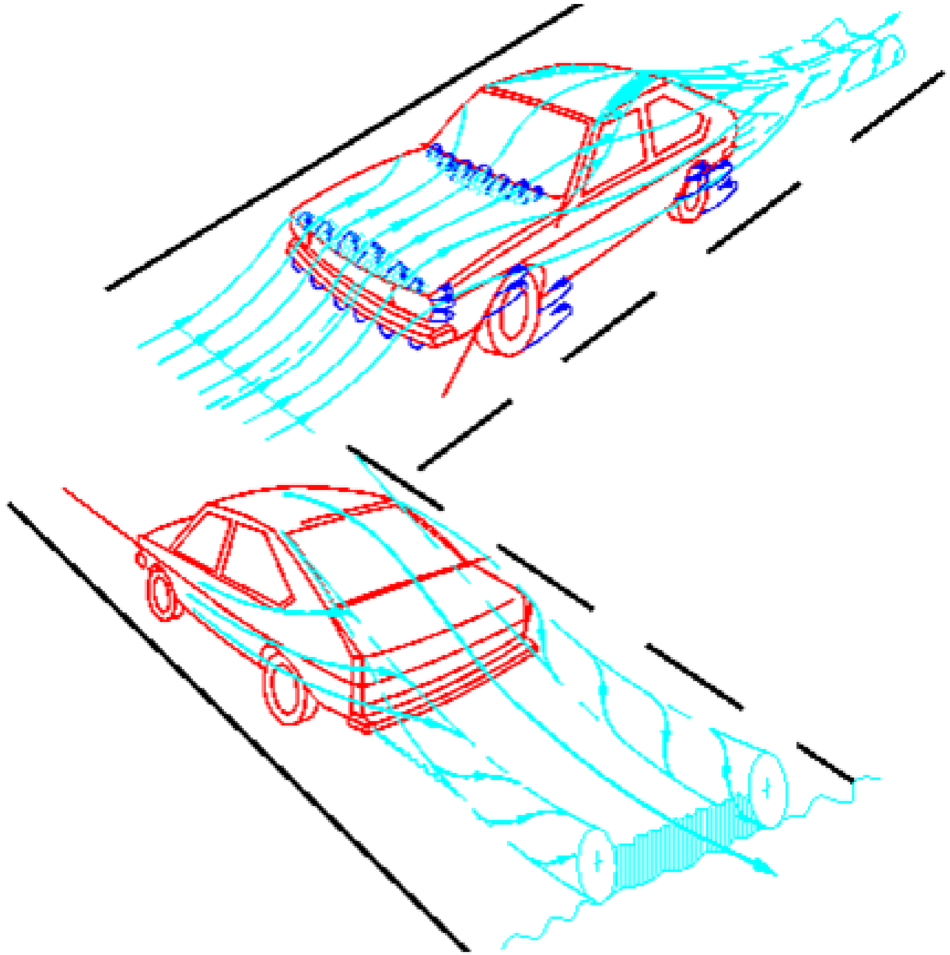

Aerodynamic drag
Friction drag
Pressure drag
Trailing vortex trag



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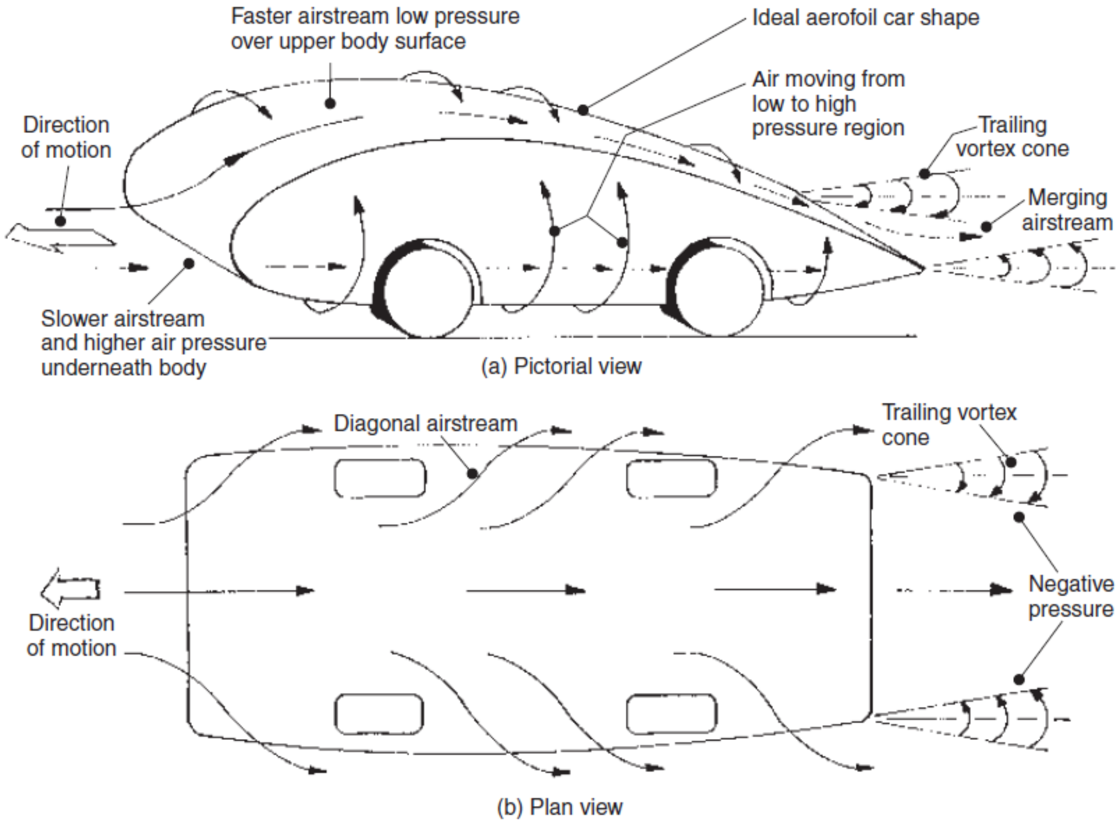
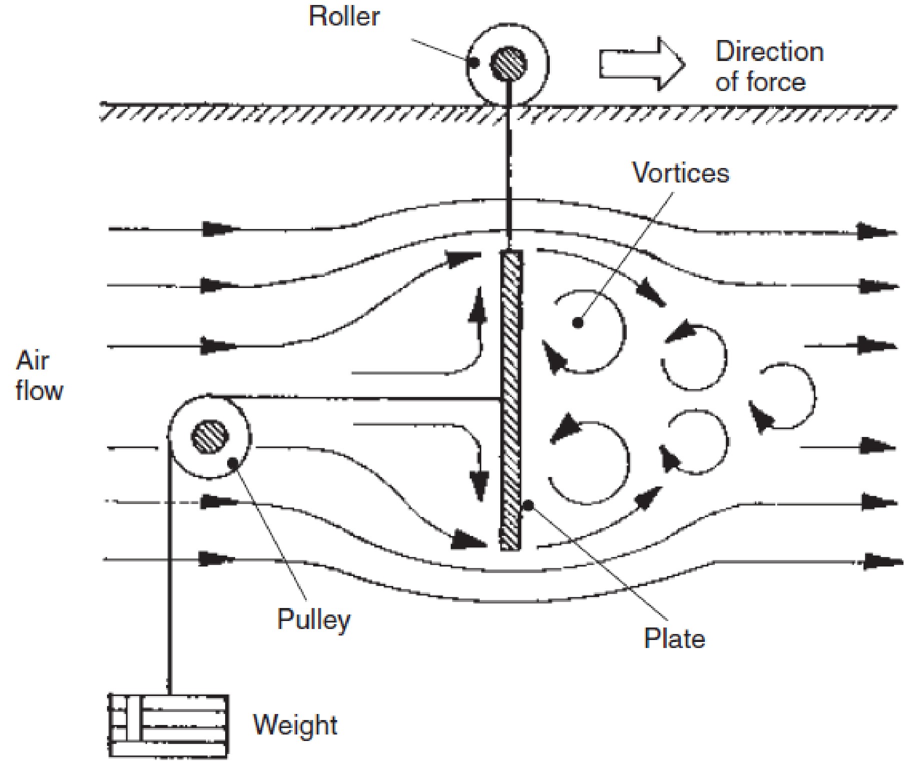
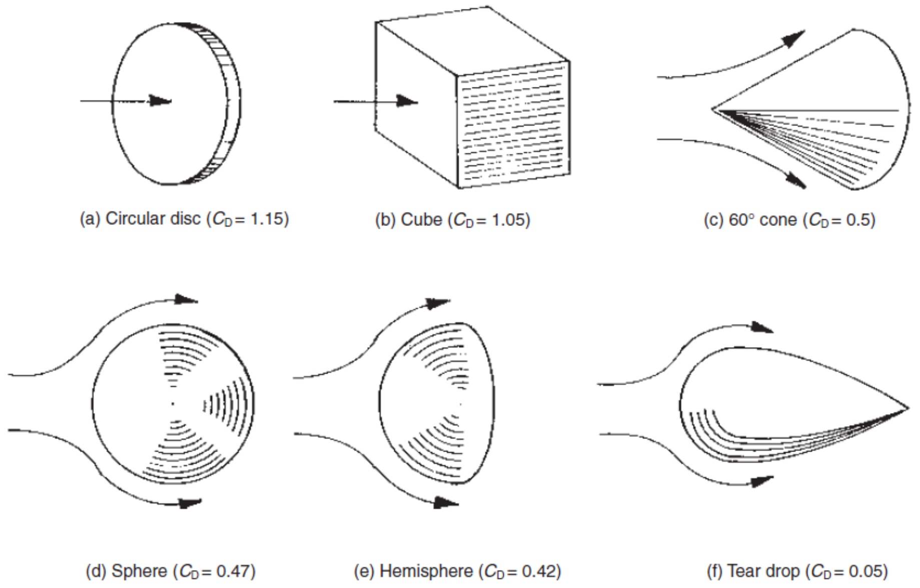


HEISLER, H. 2002. *Advanced Vehicle Technology*, Butterworth-Heinemann.

Pressure drag

- $F = \rho A V^2, N$
- $F \propto A V^2, N$
- $F = C_D A V^2, N$

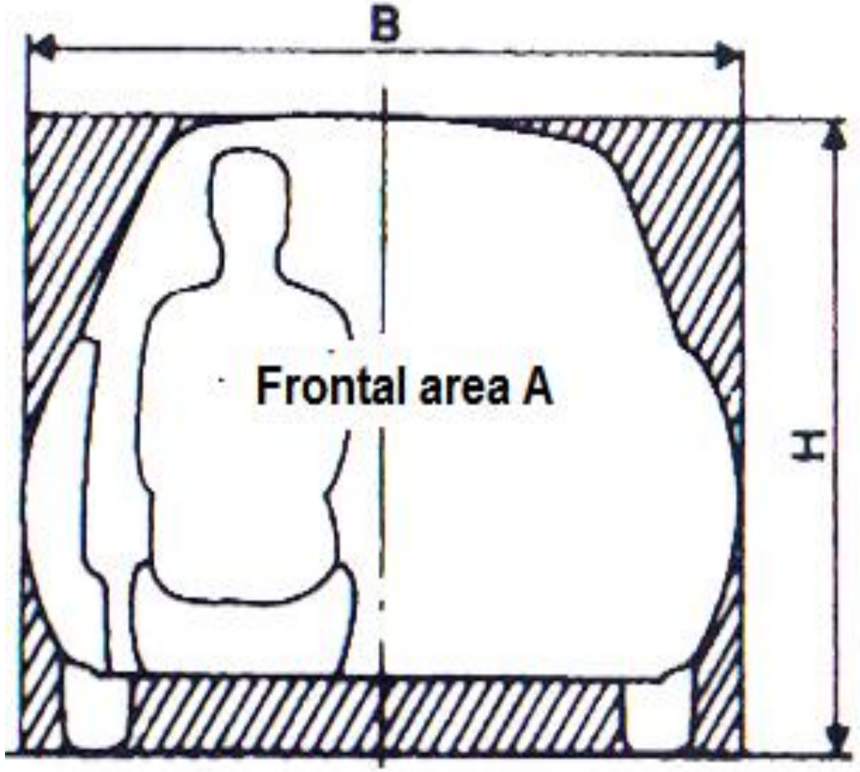


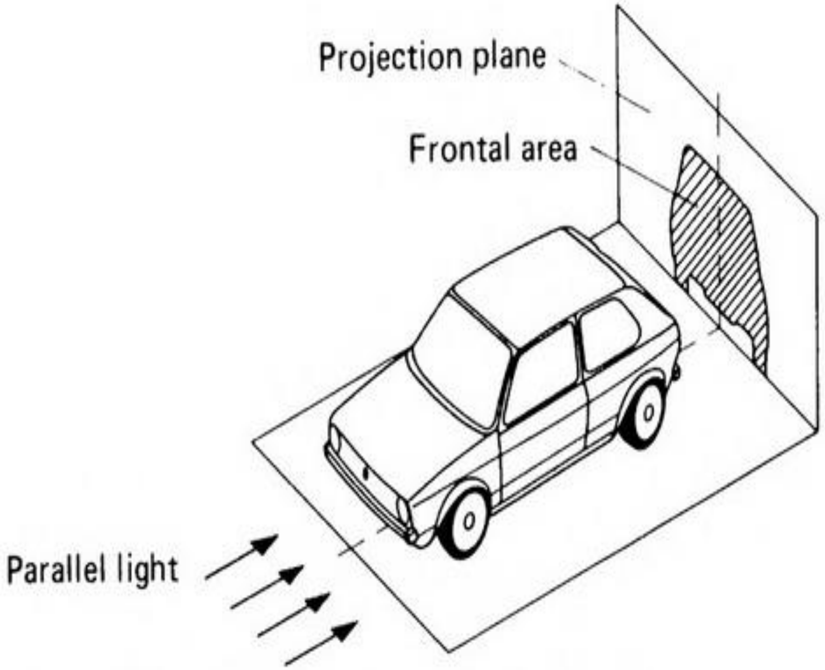


HEISLER, H. 2002. *Advanced Vehicle Technology*, Butterworth-Heinemann.

Frontal area

$$A = w_p H B$$





Aerodynamic forces & moments

Forces

- Integration of pressure field and friction effects produce an aerodynamic force acting at the centre of pressure (COP)
- (COP) is a theoretical point of force application and usually it is distant from the centre of gravity

Moments

- moments reference centre is located in the middle between front and rear axle or the **centre of gravity**

Aerodynamic forces

all aero forces are strongly dependent on car’s body construction and the wind direction

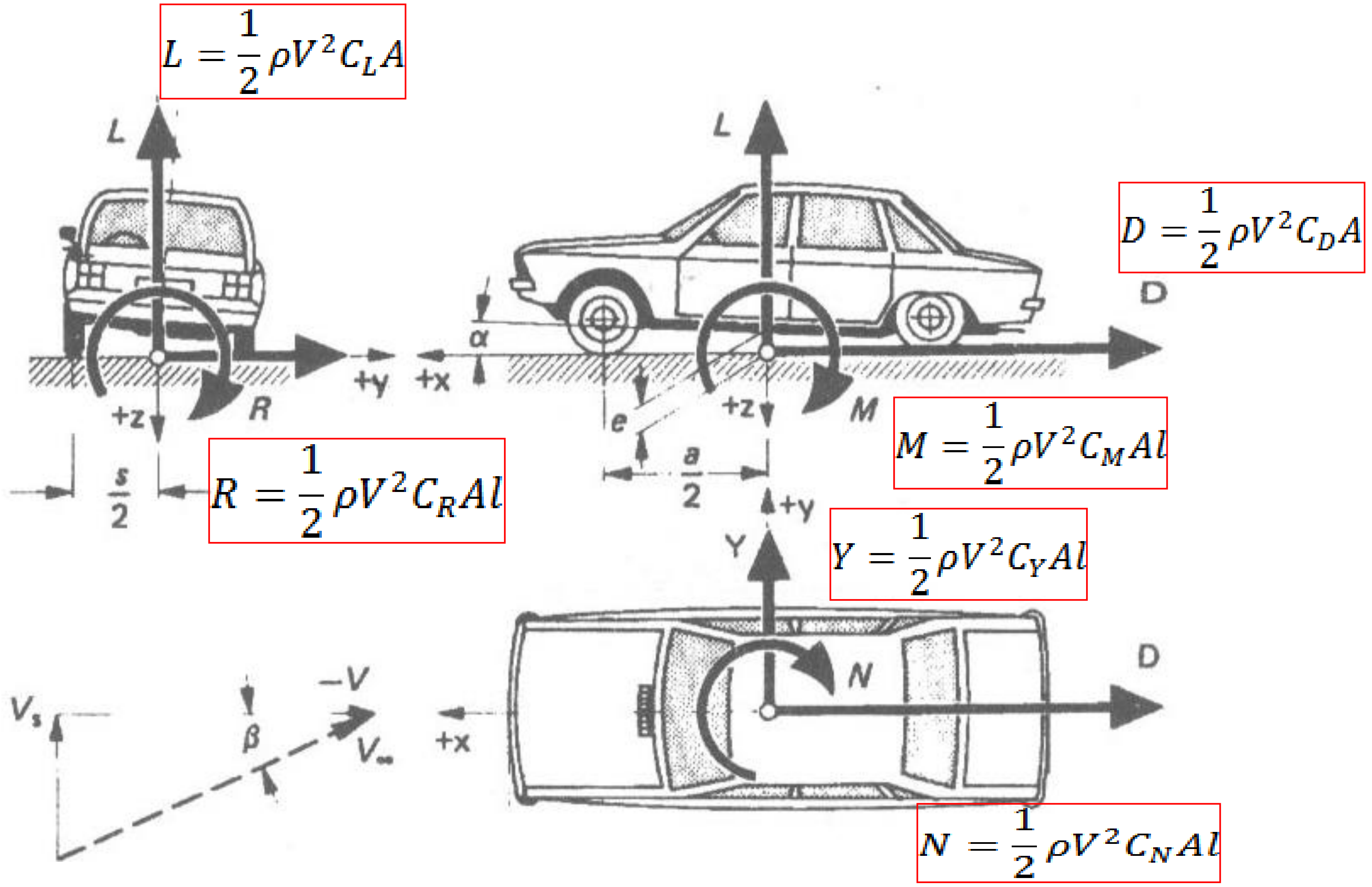
Crosswind β≠0

- Side force
- Yawing and rolling moment
- Rolling moment

Direct flow β=0

- Drag force
  - Friction drag
  - Pressure drag
  - Trailing vortex drag
- Lift force
- Pitching moment

The aerodynamic forces



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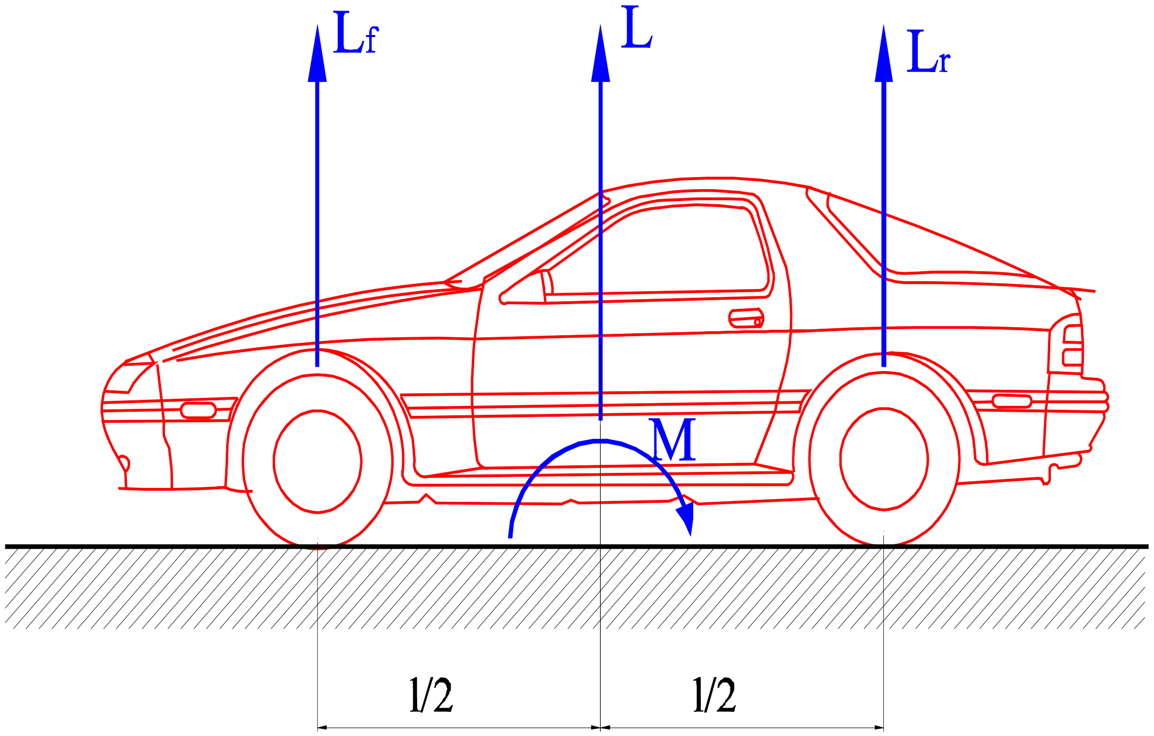
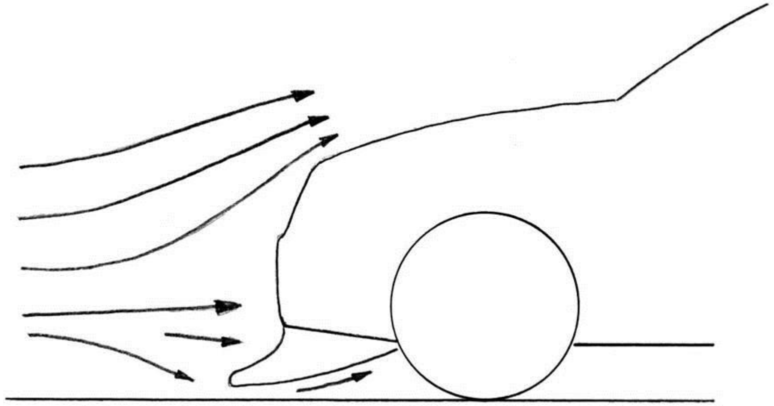
Lift and pitching moment

- $L = L_F + L_R$
- $-\frac{1}{2}L_F + M + \frac{1}{2}L_R = 0$
- $M = \frac{1}{2}L_R - \frac{1}{2}L_F$
- $C_{L_F} = \frac{1}{2}C_L + C_M$
- $C_{L_R} = \frac{1}{2}C_L - C_M$



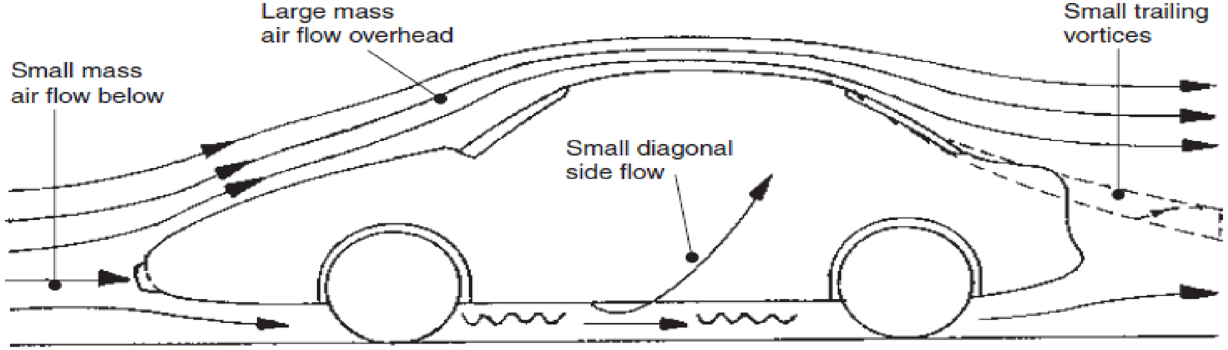



NOVOTEL

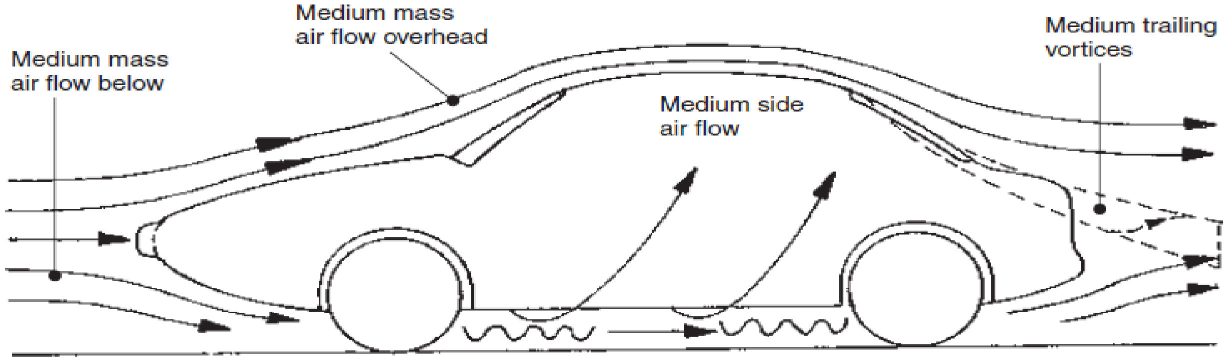

HOLDEN
HOLDEN
HSV.COM.AU
Mobil 1
NGK


BLAUPUNKT
AMG
Mobil
DUNLOP

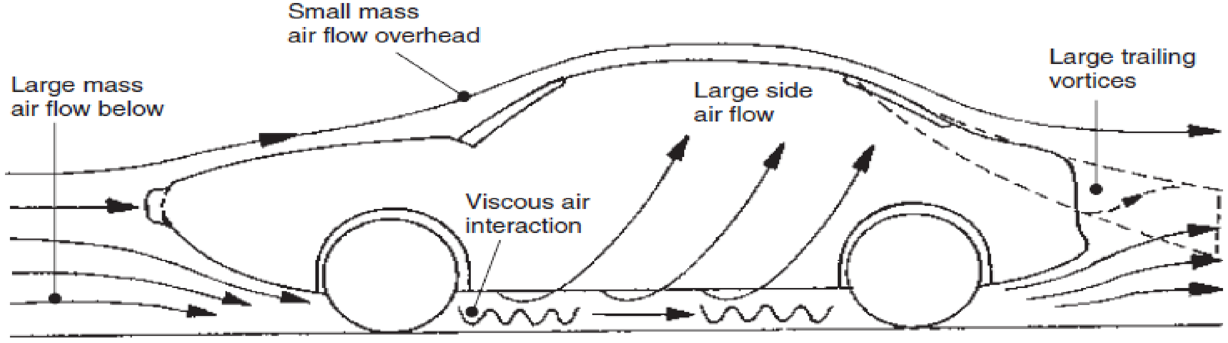

BLAUPUNKT
Audi Bank

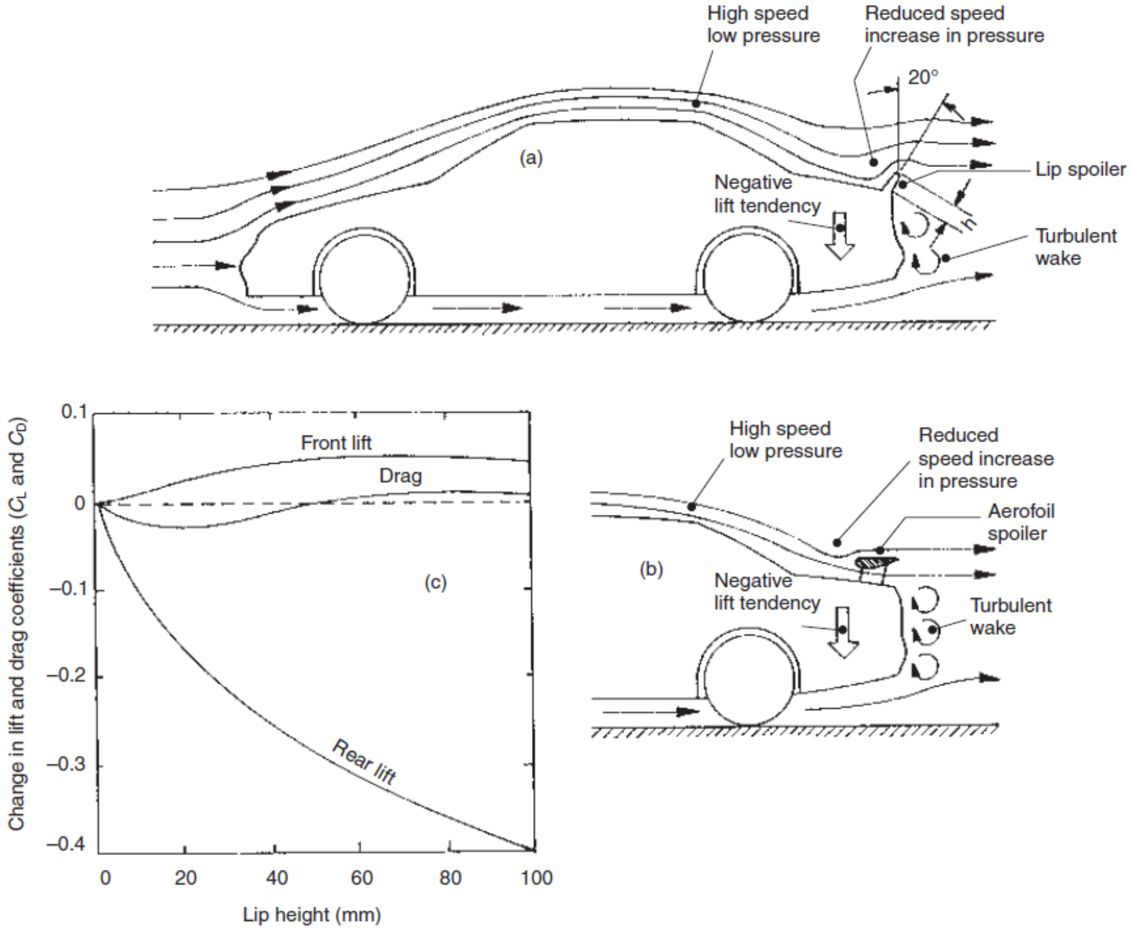


(a) Downturned nose profile

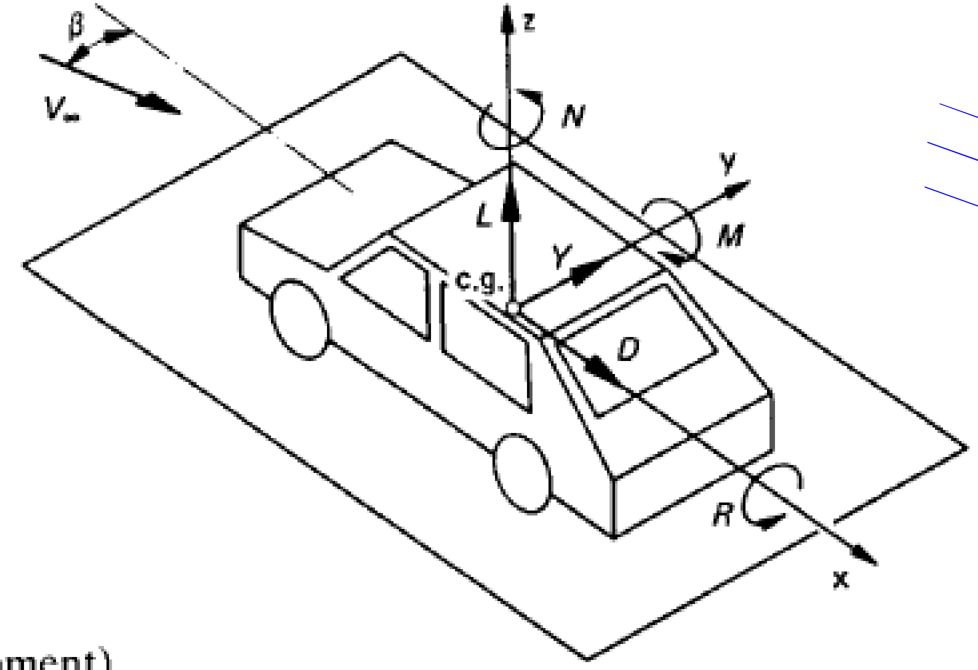


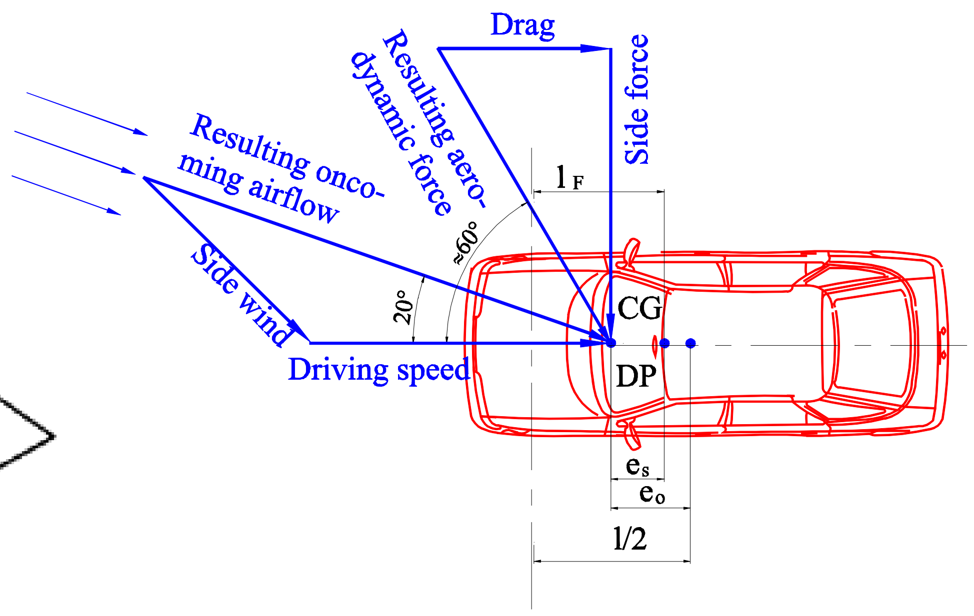
(b) Central nose profile





Cross wind





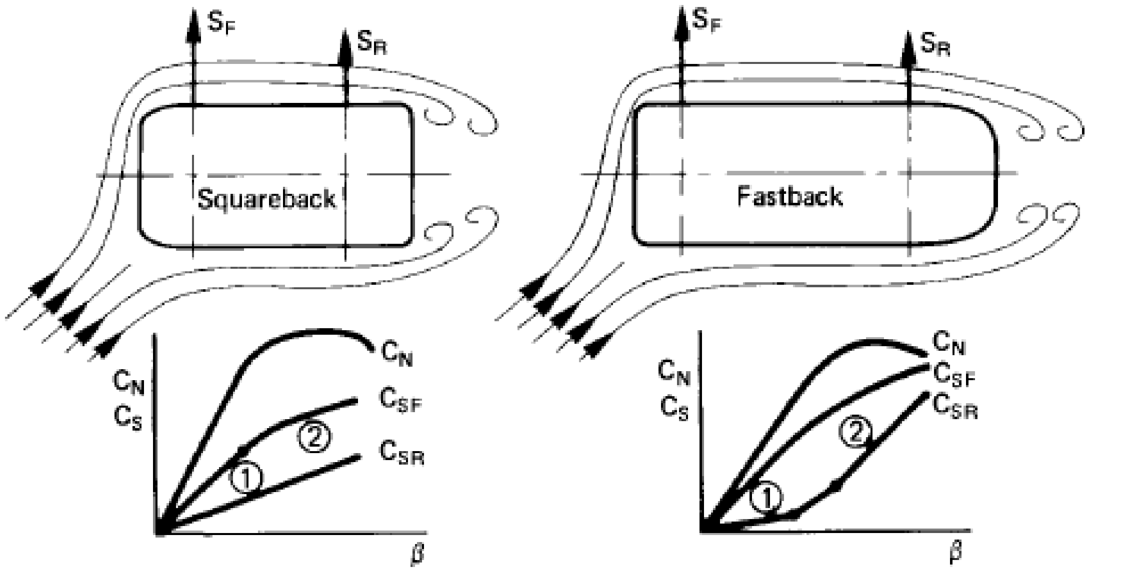
$$c_D = \frac{D}{\frac{\rho}{2} V_\infty^2 A} \quad \text{(drag)}$$

$$c_M = \frac{M}{\frac{\rho}{2} V_\infty^2 A l} \quad \text{(pitching moment)}$$

$$c_Y = \frac{Y}{\frac{\rho}{2} V_\infty^2 A} \quad \text{(side force)}$$

$$c_R = \frac{R}{\frac{\rho}{2} V_\infty^2 A l} \quad \text{(rolling moment)}$$

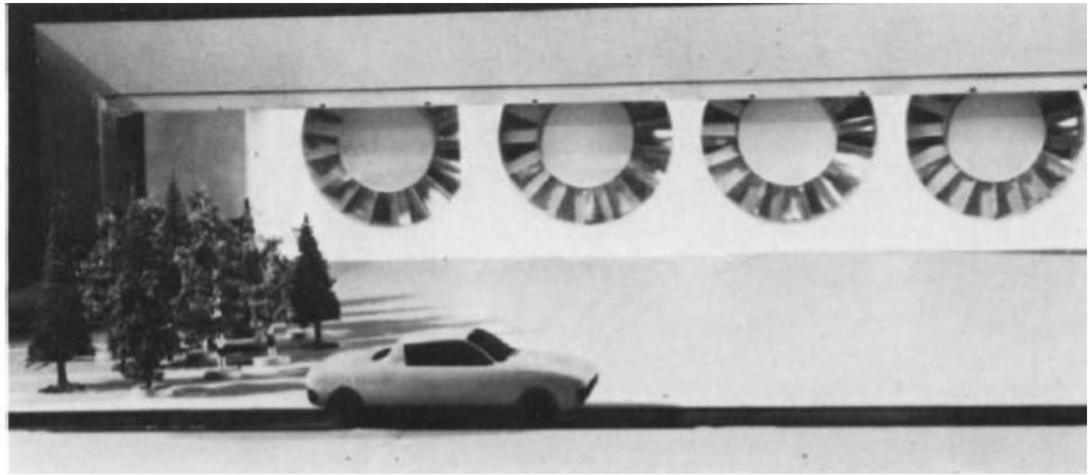
$$c_N = \frac{N}{\frac{\rho}{2} V_\infty^2 A l} \quad \text{(yawing moment)}$$

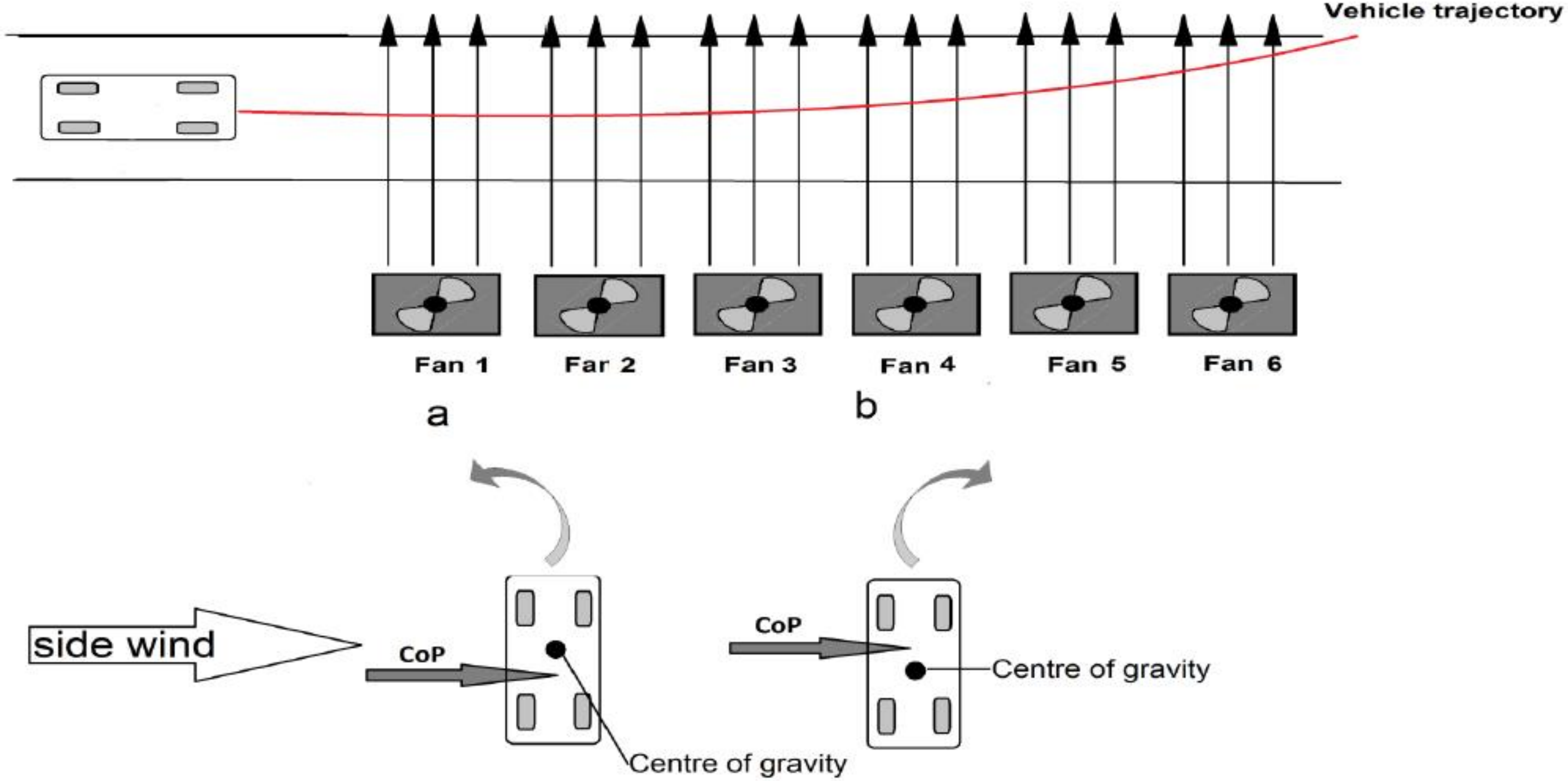


① Attached leeward front airflow.
② Separated leeward front airflow.

① Airflow is attached in C pillar area.
② Completely separated leeward rear end airflow.

Crosswind sensitivity



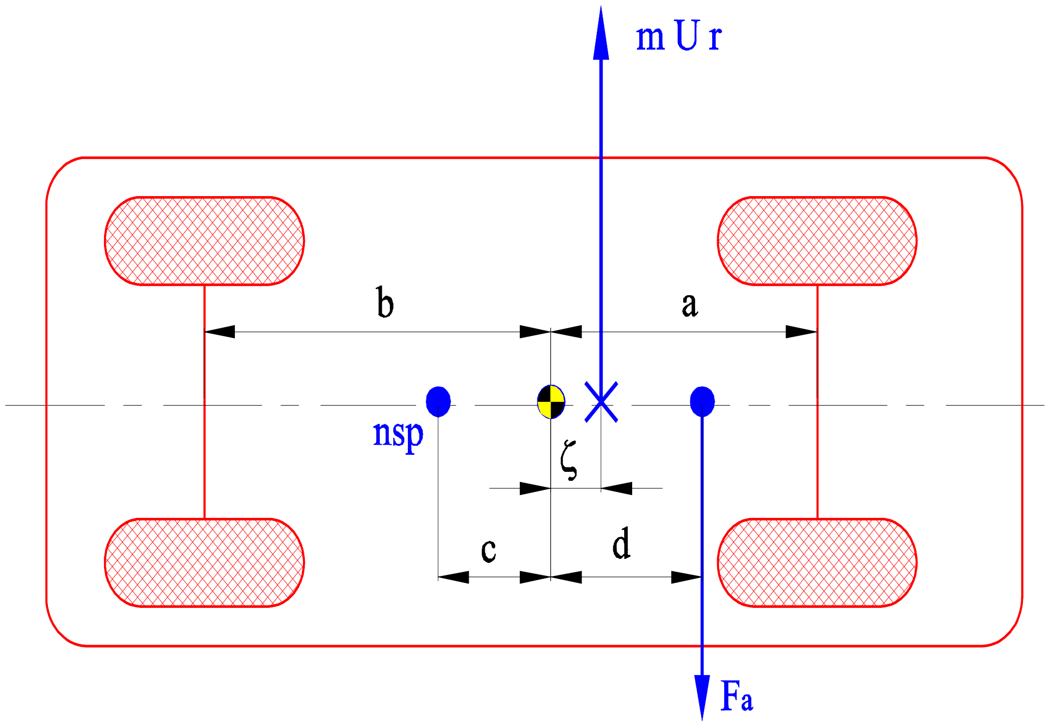


Crosswind sensitivity

- $$\frac{r}{\propto} = \frac{\frac{\frac{1}{2}\rho V^2 C_S A}{mU} \cdot \left[c + \left(a - \frac{L}{2} + \frac{L C_Y}{C_S}\right)\right]}{c + \xi}$$

Where :

- **a** is a distance from mass centre to front axle
- **b** distance from mass centre to rear axle
- **nsp** is a neutral steer point ("centre of tire forces")
- **m** is a vehicle mass
- **CoP** is a aerodynamic centre of pressure
- **c** is a distance from nsp forward to mass centre = $(bC_r - aC_f) / (C_f + C_r)$
- **d** is a distance from mass centre forward to CoP
- **U** is a vehicle speed
- **V** is speed of wind generated by the fans
- **r** is a vehicle steady turning yaw rate response
- **S** is a constant, aerodynamic side force
- **ζ** is a moment arm proportional to the tire force yaw damping moment about the nsp($\zeta = (a+b)^2 C_f C_r / (C_f + C_r) mU^2$)
- **α** is a slip angle
- **C_f** effective total tire cornering stiffness of front axle
- **C_r** effective total tire cornering stiffness of rear axle
- **C_Y** is a aerodynamic yaw moment coefficient
- **C_S** is a side force coefficient
- **L** is a wheel base of a vehicle ($L = a+b$)
- **A** is a frontal area of a vehicle



MACADAM, C. C., SAYERS, M. W., POINTER, J. D. & GLEASON, M. 1990. Crosswind Sensitivity of Passenger Cars and the Influence of Chassis and Aerodynamic Properties on Driver Preferences. *Vehicle System Dynamics,* 19, 201-236.