Experiment 2. Practical implementation of adiabatic process

1 Introduction

The process is adiabatic when there is no heat exchange between the thermodynamic system and the surrounding $(Q_{1-2} = 0)$. From this definition it follows that to implement the adiabatic process, e.g. gas expansion in a cylinder with a movable piston, the cylinder and the piston must be covered by the perfect heat insulator.

Analogously, if a tank filled with gas (air) is emptied by opening the valve, the state of the gas in the tank changes according to adiabatic process ONLY when walls of the tank were perfectly thermally insulated. Because there is no perfect insulation, in practice, we can obtain an adiabatic process with some accuracy (never perfect adiabatic process). The indicator below is a measure of this approximation

$$Y = \frac{|Q_{z1-2}|}{|U_2 - U_1|} \tag{1}$$

Where:

 $|Q_{z1-2}|$ - the total amount of heat (positive or negative) of the gas during $\Delta \tau$, $|U_2 - U_1|$ total change in internal energy of the gas at the transition from state 1 to state 2. If Y = O, it means that adiabate was realised. Otherwise, depending on the specific value of this indicator, it is said that the adiabatic process was realised with approximation defined by Y.

Finding the parameters required for the determination of Y is rather problematic. The problem is significantly simplified when the gas is considered as a perfect and the adiabatic equation is used as a reference (equation 2)

$$\mathbf{p} \, \boldsymbol{\upsilon}^{\,\mathbf{k}} = \mathbf{idem} \tag{2}$$

Where:

k is an adiabatic constant, given by:

$$k = \frac{c_p}{c_v} \tag{2.1}$$

The process is considered as adiabatic only for specific values of the exponent -i.e. 1.4 for air.

2 The aim of the experiment

The aim of the experiment is:

- verify if the decompression of the air in tank is (approximate) an adiabatic process,
- calculate the accuracy of the adiabatic process

3 Description of the experiment



Figure 1. Test stand, [1] tank A, [2] tank B, [3] pressure gauge, [4] connection valve A with B, [5] pump, [6] valve, [7] key

1. Containers A and B with constant volume V should be inflated with air until the overpressures Δp_{A1i} and Δp_{B1i} (indicated in the measurement sheet) are adequately obtained.

$$\mathbf{p}_{A1i} > \mathbf{p}_{B1i} \ge \mathbf{p}_0 \tag{3}$$

Where:

 p_o ambient pressure

$$p_{A1i} = p_o + \Delta p_{A1i}$$

$$p_{B1i} = p_o + \Delta p_{B1i}$$

The gas temperature in the tanks after the end of pumping should be equal to ambient temperature, i.e.:

$$\mathbf{t}_{\mathbf{A}\mathbf{1}\mathbf{i}} = \mathbf{t}_{\mathbf{B}\mathbf{1}\mathbf{i}} = \mathbf{t}_{\mathbf{0}} \tag{4}$$

2. Open the valve connecting tanks A and B for approx. 1 second. In such a case, a rapid flow between the containers occurs, which ends when the pressure equalises. i.e.

$$\mathbf{p}_{A2i} = \mathbf{p}_{B2i} = \mathbf{p}_{mi} > \mathbf{p}_0 \tag{5}$$

The temperatures reach values of:

$$\mathbf{t}_{A2i} < \mathbf{t}_0, \, \mathbf{t}_{B2i} > \mathbf{t}_0 \tag{6}$$

Note:

The average pressure (p_m) is not measured because for a perfect gas it can be calculated from the formula:

$$p_m = \frac{p_{A1} + p_{B1}}{2} \tag{7}$$

Derivation of the formula:

In the time interval between valve opening and closing, no external work is performed, and the heat emission is negligible due to the high speed of the process. Therefore, it can be assumed that the total internal energy of the system does not change (however, it is only valid when the temperatures are in equilibrium – as explained in equation 4)

For a perfect gas, the internal energy is given by the formula (8):

$$U = \frac{pV}{k-1} + U_0 \tag{8}$$

Therefore, the condition of energy conservation for the system is expressed by the equation:

$$\frac{p_{A1}V}{k-1} + \frac{p_{B1}V}{k-1} = \frac{p_{A2}V}{k-1} + \frac{p_{B2}V}{k-1}$$
(9)

By substituting $p_{A2} = p_{B2} = p_m$ the desired equation is obtained.

3. After closing the valve wait until the air temperature in the tanks equalises with ambient (equation 4)

Then the Δp_{A3i} pressure can be read, and the p_{A3i} value can be calculated:

The above-described activities should be repeated for several different pressures p_{Bi} and the same initial pressure value p_{Ai} .



Figure 2. Decompression process in container A

The following dependencies apply:

$$\begin{split} & T_{2} < T_{o} \qquad p_{A2i} = p_{mi} \quad (i = 1, 2,, 5) \\ & T_{3} = T_{o} \qquad p_{A3i} \rangle p_{m} \\ & p_{0} \upsilon_{0} = RT_{0} \qquad (R = 0.287 \text{ kJ/kg K}) \\ & p_{mi} = \frac{p_{A1i} + p_{B1i}}{2} \\ & p_{A1i} = p_{o} + (\Delta p)_{A1i} \qquad p_{B1i} = p_{0} + (\Delta p)_{B1i} \end{split}$$

$$(\Delta p)_{A1i} = (\Delta p)_{A1}$$

-constant, e.g. 800 mm H₂0

 $(\Delta p)_{B1i}$ - 0, 150, 300, 450, 600, mm H₂0 $p_{mi} = \frac{p_o + (\Delta p)_{A1} + p_o + (\Delta p)_{B1i}}{2}$ Or

$$p_{mi} = p_o + \frac{(\Delta p)_{A1} + (\Delta p)_{B1i}}{2}$$
$$p_{A3i} = p_o + (\Delta p)_{A3i}$$

The gas states "1", "2" are on the polytrop, therefore they fulfil the equation $p v^w = idem$:

$$p_{A1}\upsilon_{A1}^w = p_{A2i}\upsilon_{A2i}^w = p_{mi}\upsilon^w$$

Hence:

$$\frac{p_{A1}}{p_{A2i}} = \left(\frac{\upsilon_{A2i}}{\upsilon_{A1}}\right)^{w}$$

The states "1" and "3" of gas lies on the isotherm of $t = t_0$. Therefore, they satisfy the equation:

 $p_{A1}v_{A1} = p_{A3i}v_{A3i}$
Since :

 $v_{A3i} = v_{A2i}$

then:

$$p_{A1}v_{A1} = p_{A3i}v_{A2i}$$

 $\frac{v_{A2i}}{v_{A1}} = \frac{p_{A1}}{p_{A3i}}$

$$\frac{p_{A1}}{p_{A2i}} = \left(\frac{p_{A1}}{p_{A3i}}\right)^W$$
 and $\frac{p_{A1}}{p_{A3i}} = \left(\frac{p_{A1}}{p_{A3i}}\right)^W$

Logarithm results in:

$$ln\frac{p_{A1}}{p_{mi}} = wln\frac{p_{A1}}{p_{A3i}}$$

Which can be further expressed as:

$$\eta_i = w \xi_i$$

4. Results elaboration

$(\Delta p)_{Ai}$		To set				
$(\Delta p)_{B1i}$	0	150	300	450	600	To set
$(\Delta p)_{A3i}$						To measure
P _{mi}						To calculate
P_{Ai}						To calculate
P _{A3i}						To calculate

1. The results of the measurements placed in the table

2. Calculate

$$\eta_i = \ln \frac{P_{A1}}{P_{mi}}$$

$$\xi_i = \ln \frac{P_{A1}}{P_{A3i}}$$

3. Insert the calculated values into the table and present on the graph:

$(\Delta p)_{B1i}$	0	150	300	450	600
η_i					
ξi					

- 4. Determine the trend line in the form $\eta = w\xi + u$ (first order polynomial).
- 5. Determine the analytical form of the trend line (equation) and coefficient of determination R_2

The value "w" determined in this way is the approximation of the "k".

Determine the accuracy index Y and the measurement error b (determine the value of w, in relation to the expected value k)

Adiabatic process accuracy

As it was stated in the introduction the Y indicator determines the accuracy of the adiabatic process:

$$Y = \frac{Q_{z1-2}}{U_2 - U_1} = \frac{q_{z1-2}}{u_2 - u_1}$$

Since $w \neq k$ and w = idem (because the process is polytrophic), the heat of transformation is

 $Q_{w1-2} = m c_w (T_2 - T_1)$

and

$$c_w = c_\sigma - \frac{R}{w - 1}$$

Since $U_2 - U_1 = m \text{ cv} (T_2 - T_1)$, substituting the above for "Y", yields:

$$Y = \frac{w - k}{w - 1}$$

For air k = 1.4

Determination of the relative measurement error:

$$b = \frac{|w-k|}{k} 100 \%$$