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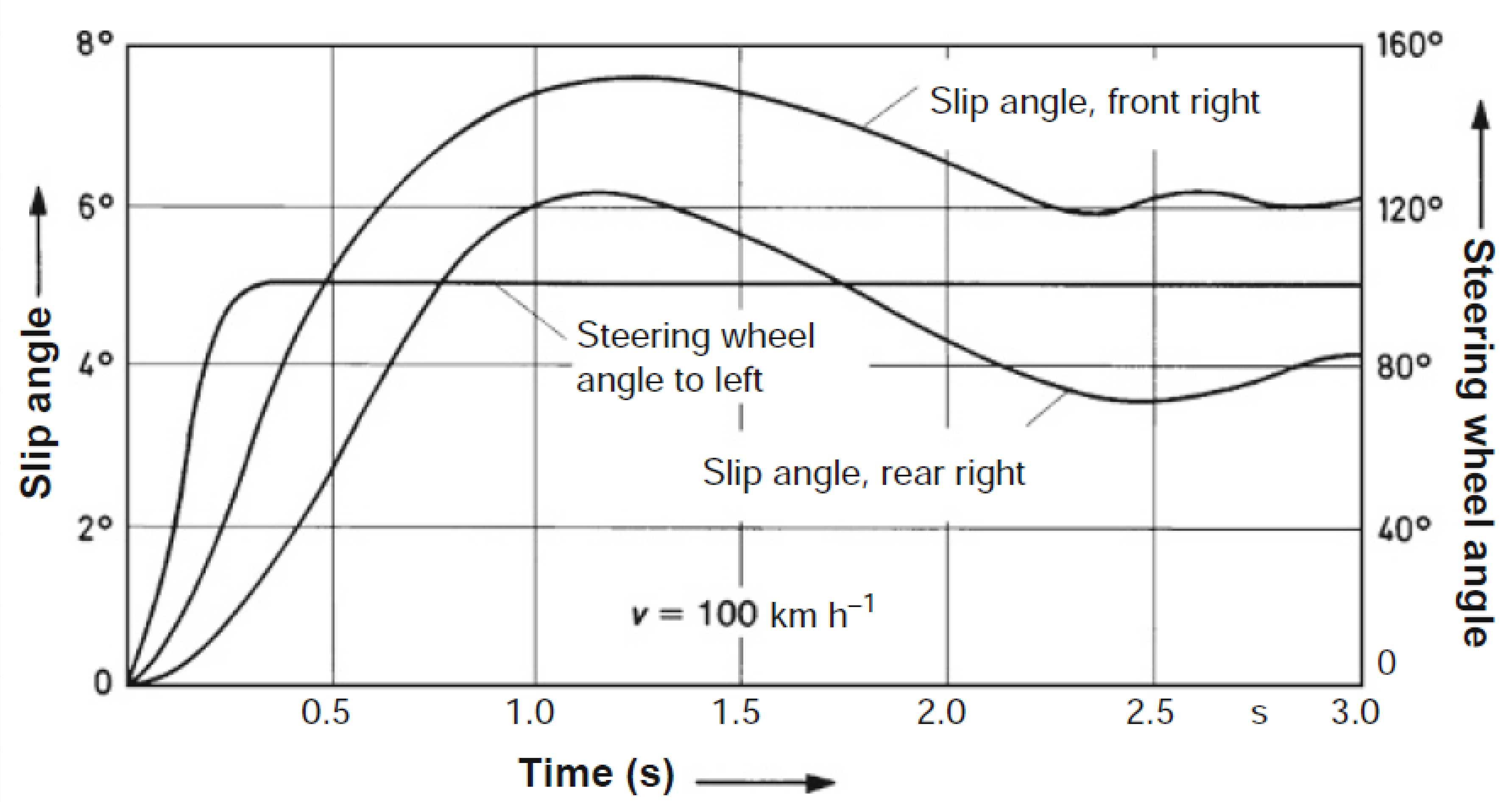
Steering systems and cornering

Steering

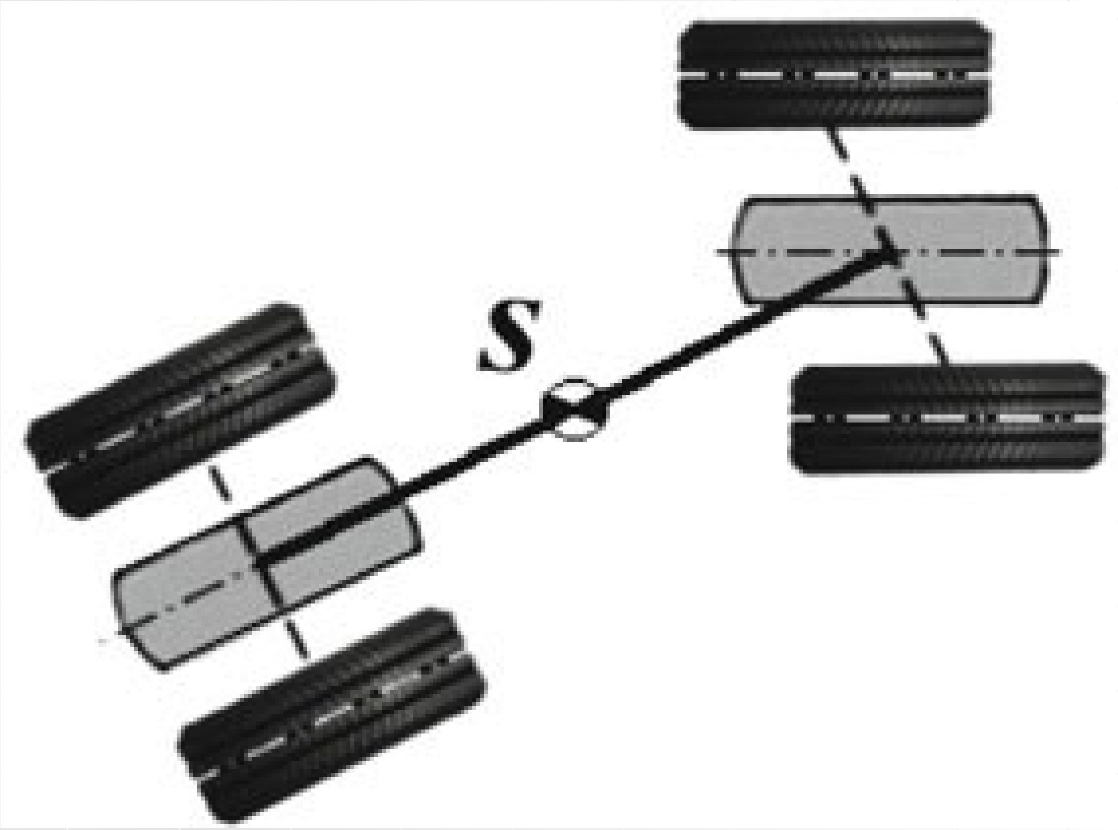
- On passenger cars, the driver must select the steering wheel angle to keep deviation from the desired course low.
- There is no definite functional relationship between the turning angle of the steering wheel made by the driver and the change in driving direction, because the correlation of the following is not linear



Simple Single-Track (Bicycle) Model

- It is assumed that the height of the vehicle’s center of gravity is at the level of the road surface.
- The equations of motion of the single-track mode. This linearization is valid for angles smaller than 4°. For larger angles, the results of the single-track model are inaccurate, since tire stiffness follows a strongly degressive curvel are linearized.
- Constant tire stiffness cα and constant vertical tire force

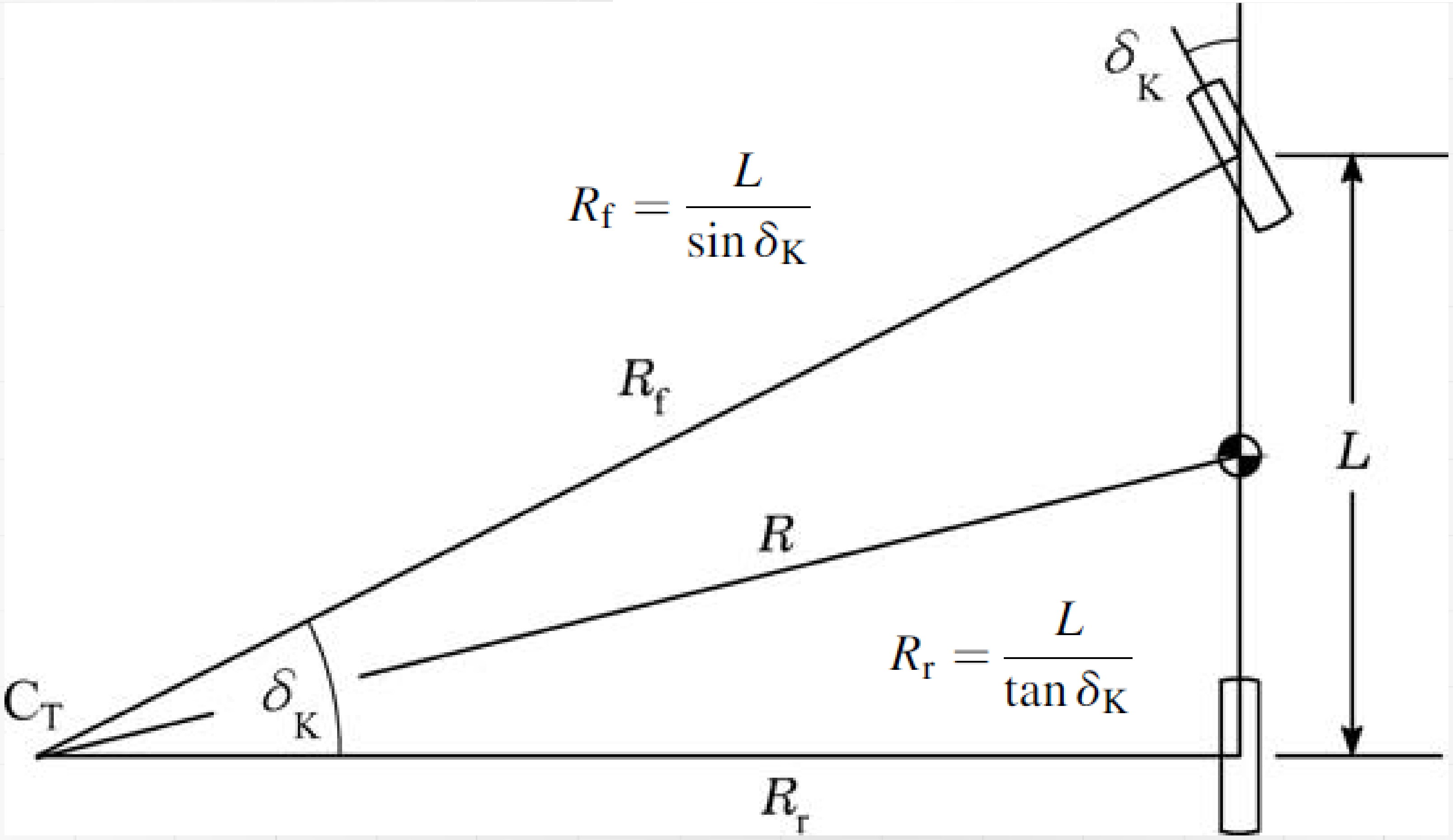
$$F_{\alpha} = c_{\alpha} \cdot \alpha$$



Turning Geometry – Single Track δ_k

the kinematic steer angle

$$\delta_K = \arctan\left(\frac{L}{R_r}\right) \approx \arctan\left(\frac{L}{R}\right) \approx \frac{L}{R}$$



Turning Geometry – Single Track

The difference of the front and rear central radii is known as the ***offtracking***.

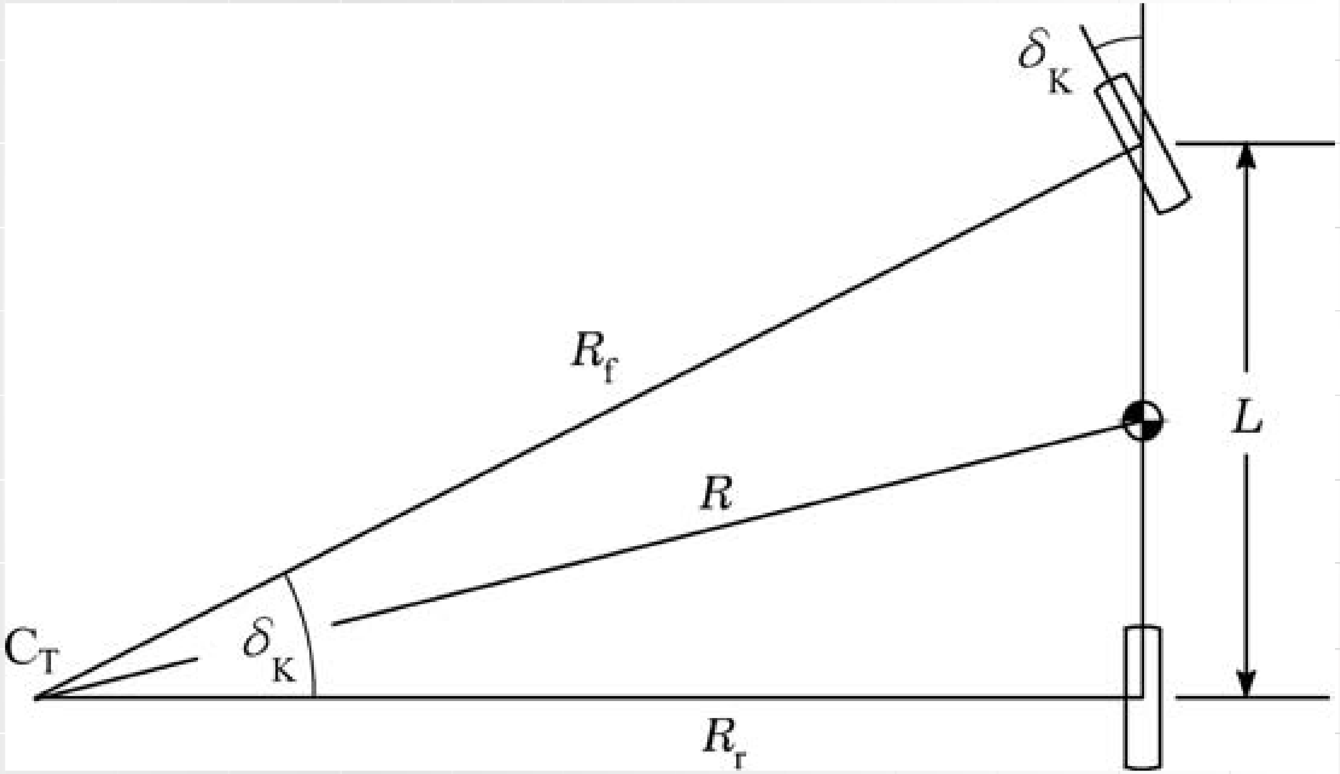
$$R_{\mathrm{OT}} = R_{\mathrm{f}} - R_{\mathrm{r}} = \frac{L}{\sin\delta} - \frac{L}{\tan\delta}$$

At low speed, with kinematic turning,

$$R_{\mathrm{f}}^2 = R_{\mathrm{r}}^2 + L^2$$

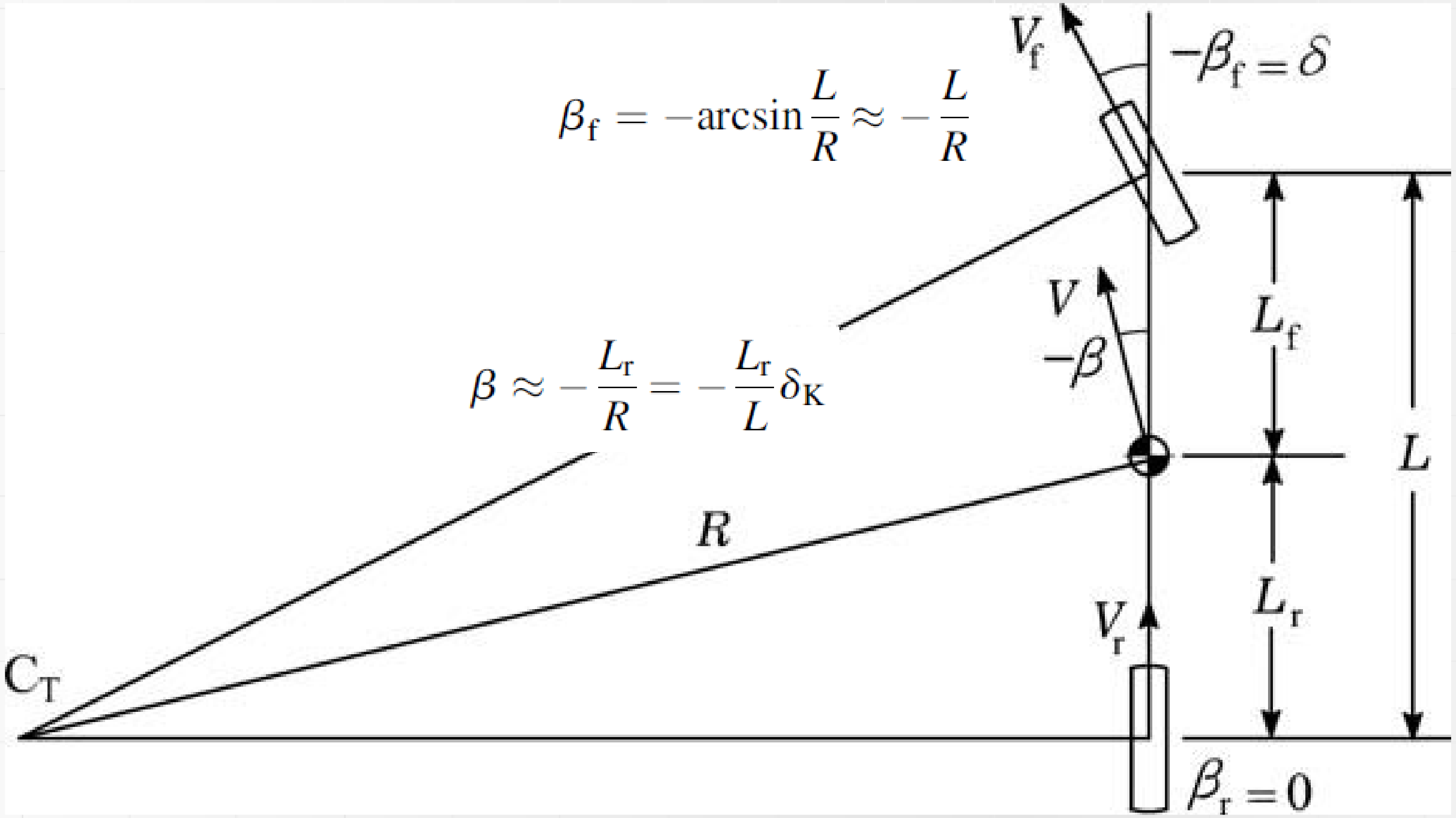
$$L^2 = R_{\mathrm{f}}^2 - R_{\mathrm{r}}^2 = (R_{\mathrm{f}} + R_{\mathrm{r}})(R_{\mathrm{f}} - R_{\mathrm{r}})$$

$$R_{\mathrm{OT}} = R_{\mathrm{f}} - R_{\mathrm{r}} = \frac{L^2}{R_{\mathrm{f}} + R_{\mathrm{r}}} \approx \frac{L^2}{2R}$$



Turning Geometry;
Attitude angles

The attitude angle β at a given point of the vehicle is the angle between the vehicle centreline and the local velocity vector, which is perpendicular to the radial lines



Attitude angle is positive with the vehicle pointing towards the inside of the corner, so the angles are negative in the figure

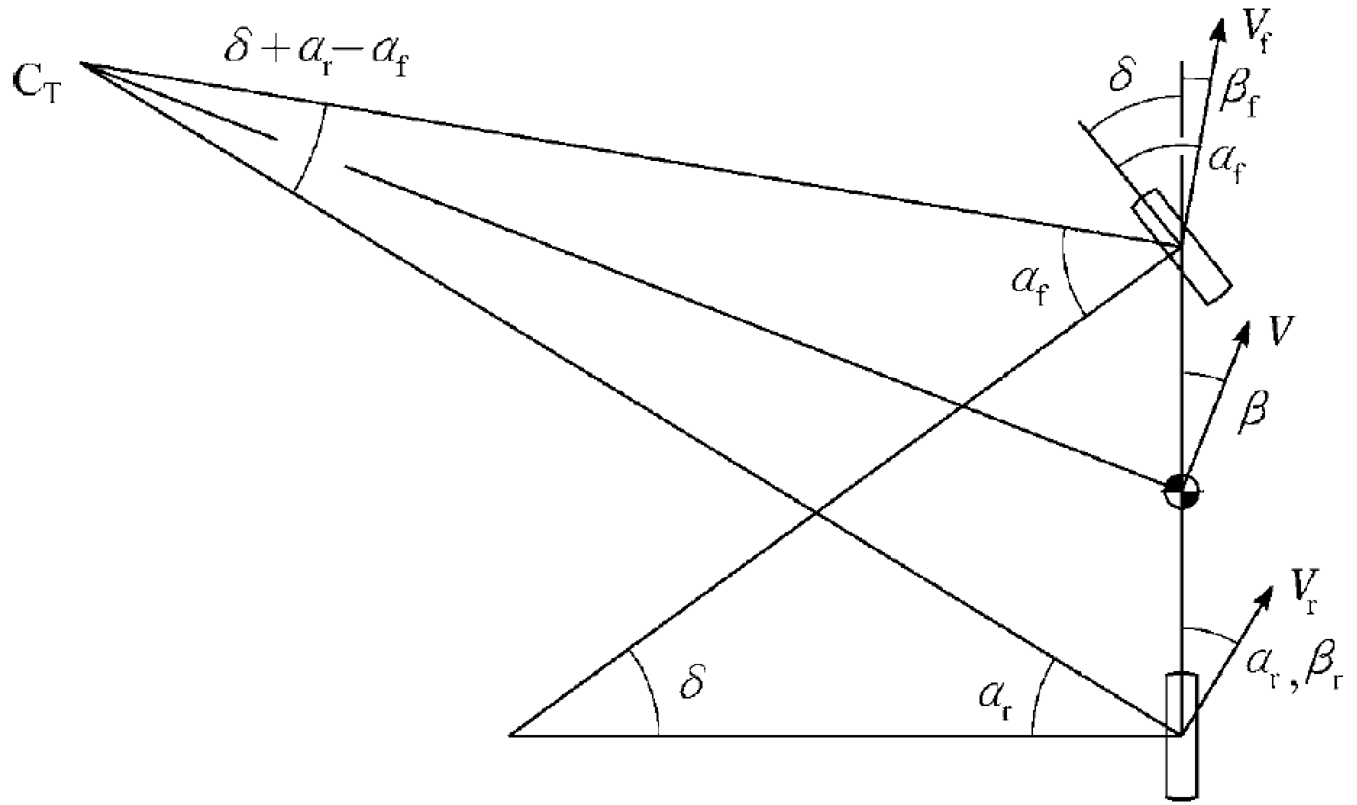
Turning Geometry;
Dynamic cornering

- tire slip angles are required to produce the necessary cornering forces - then attitude angle develops, and the offtracking also changes
- The front and rear tire slip angles are denoted by αf and αr, respectively
- The turning center point CT can be seen to have moved forward fromthe rear axle line
- or thevehicle can be considered to have rotated to a new attitude angle relative to the radius line from the turning centre

$$\beta_r = \alpha_r$$

$$\beta = -\frac{L_r}{R} + \alpha_r$$

$$\beta_f = -\frac{L}{R} + \alpha_r$$



Turning Geometry;

Ackermann Factor

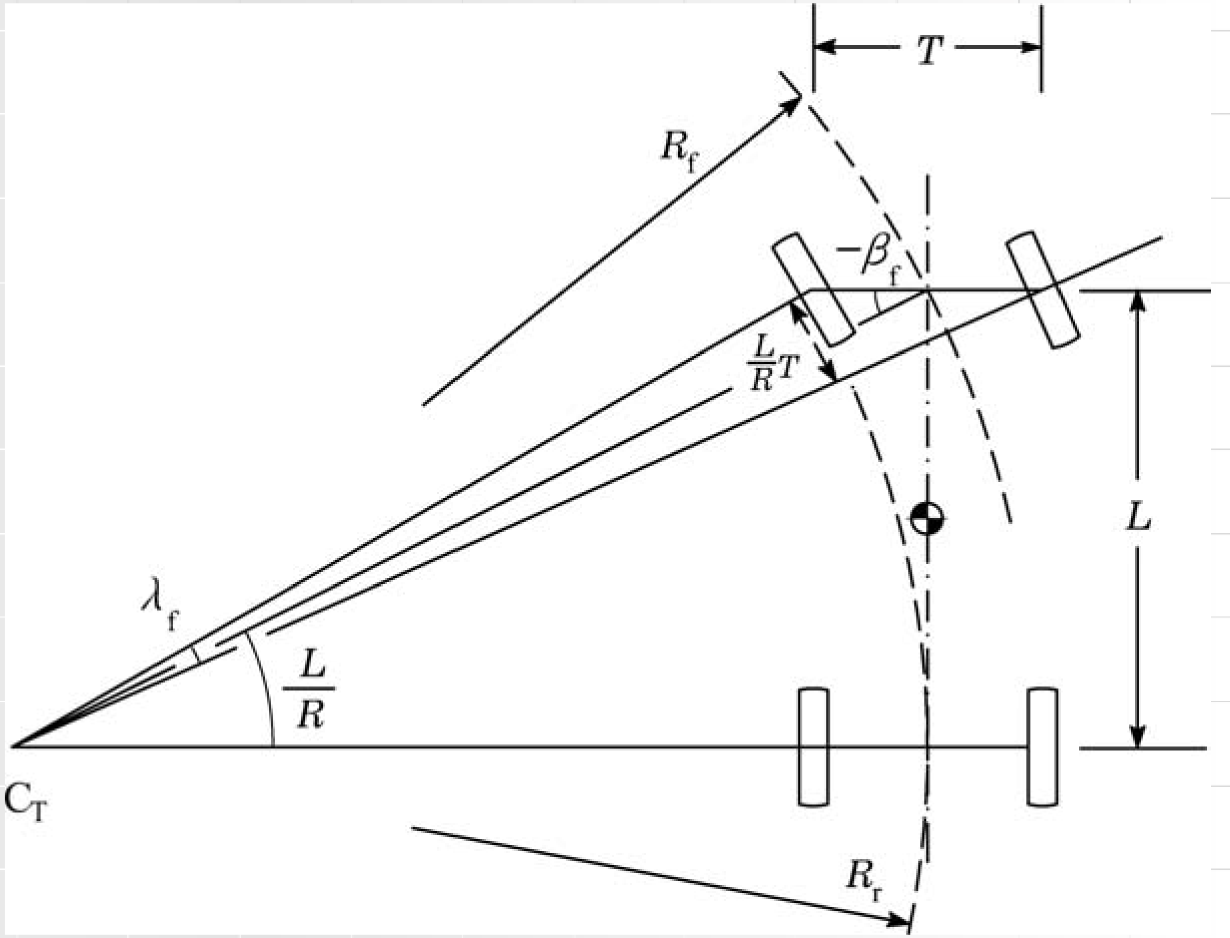
$$\lambda_{f0} = -\frac{T_f \sin\beta_f}{R_f}$$

$$\lambda_{f0} = -T_f\left(-\frac{L}{R_f}\right)\frac{1}{R_f} \approx \frac{T_f L}{R^2}$$

The actual steer angle difference occurring in a given condition, according to the actual steering mechanism geometry, is

$$\delta_{L-R} = \delta_L - \delta_R$$

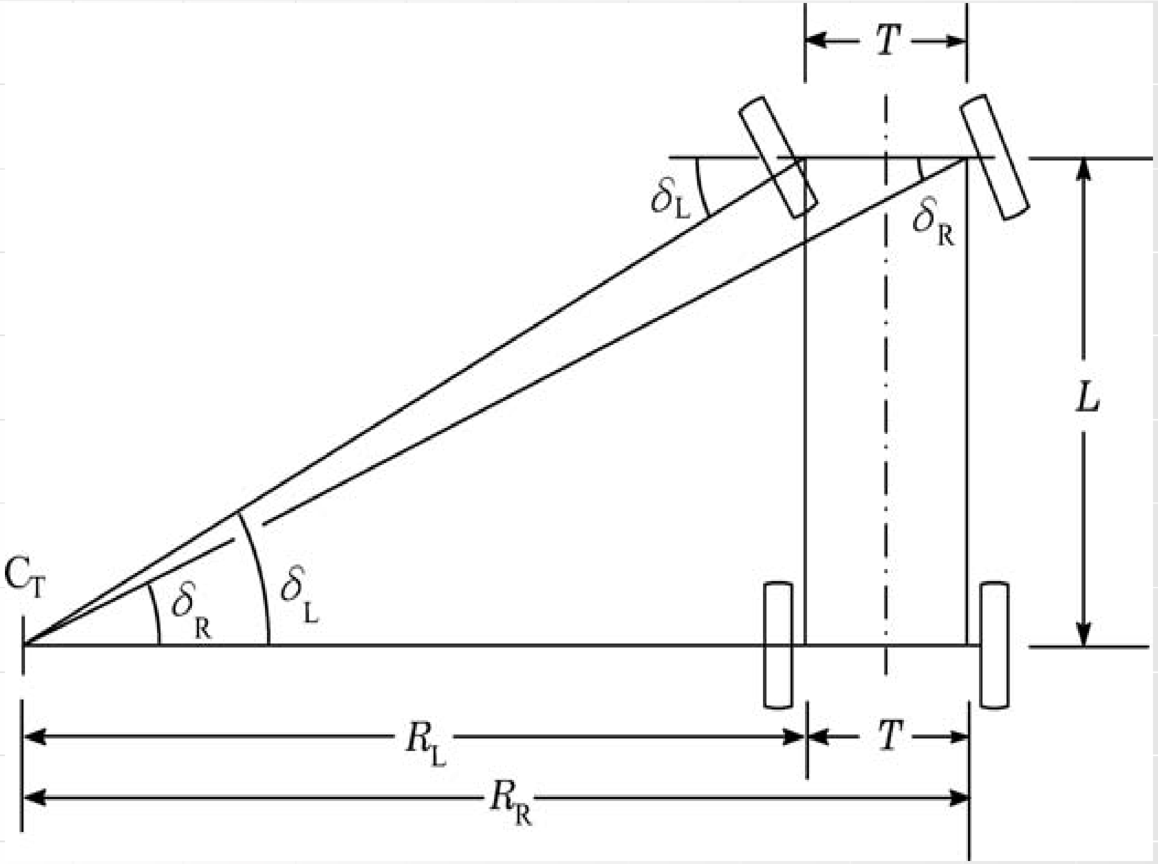
$$\delta_{L-R} = \lambda_{f0}$$

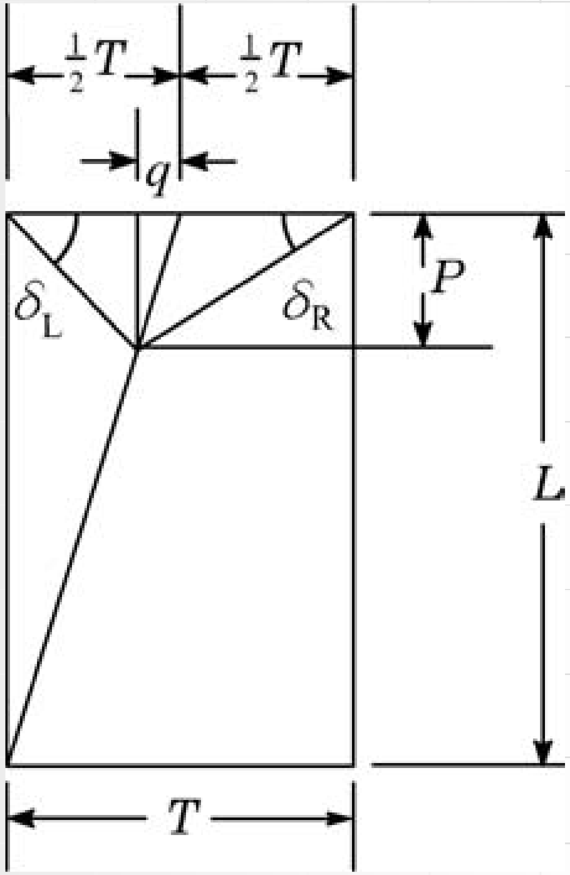


The Ackermann factor is then the ratio of the actual steer angle difference to the ideal difference

$$f_A = \frac{\delta_{L-R}}{\lambda_{f0}}$$

Geometric construction of the Ackermann condition





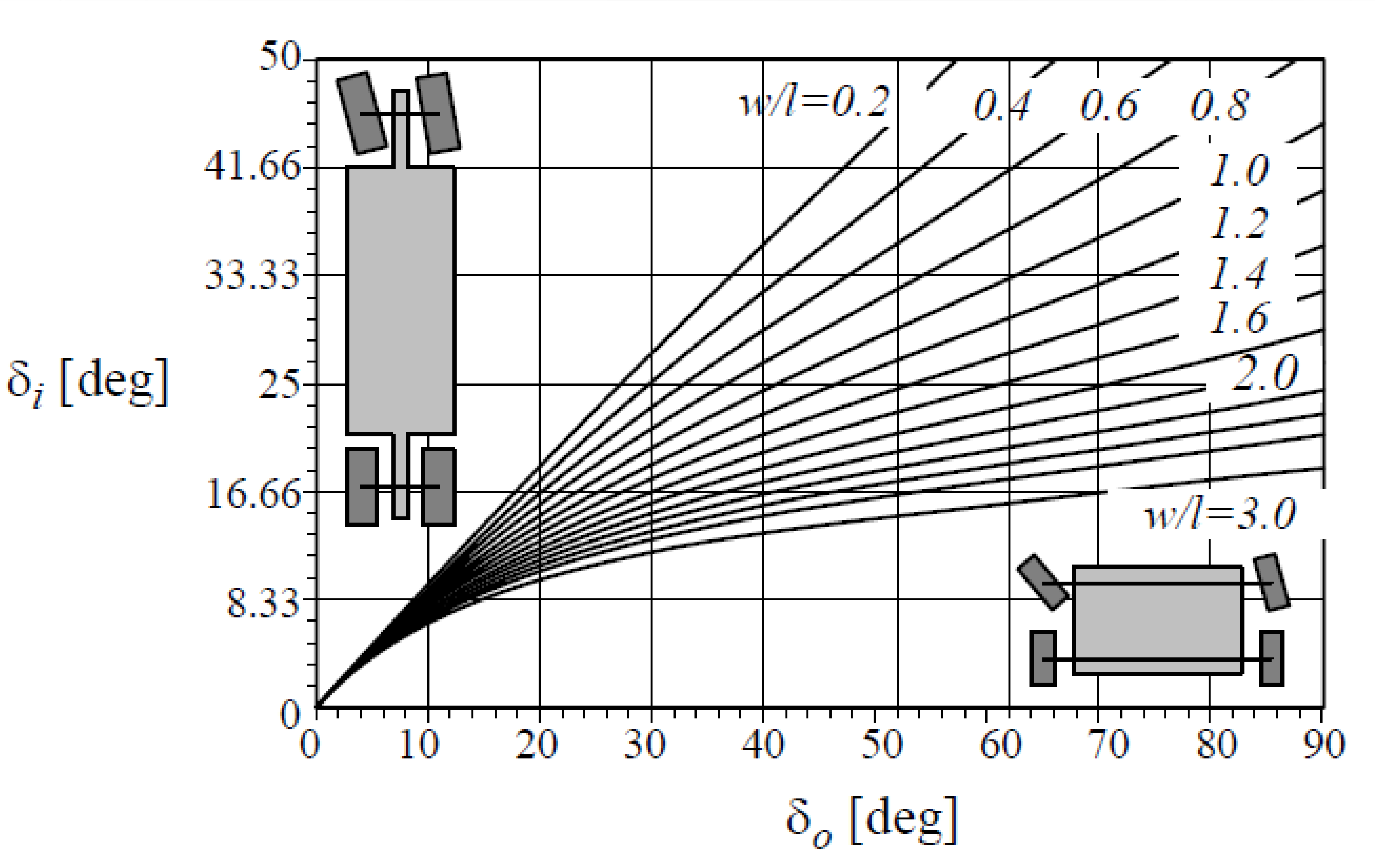
$$\cot\delta_R - \cot\delta_L = \frac{R_R}{L} - \frac{R_L}{L}$$

$$\cot\delta_R - \cot\delta_L = \frac{T}{L}$$

$$p\cot\delta_R - p\cot\delta_L = \left(\tfrac{1}{2}T + q\right) - \left(\tfrac{1}{2}T - q\right) = 2q = 2\frac{p}{L}\frac{T}{2}$$

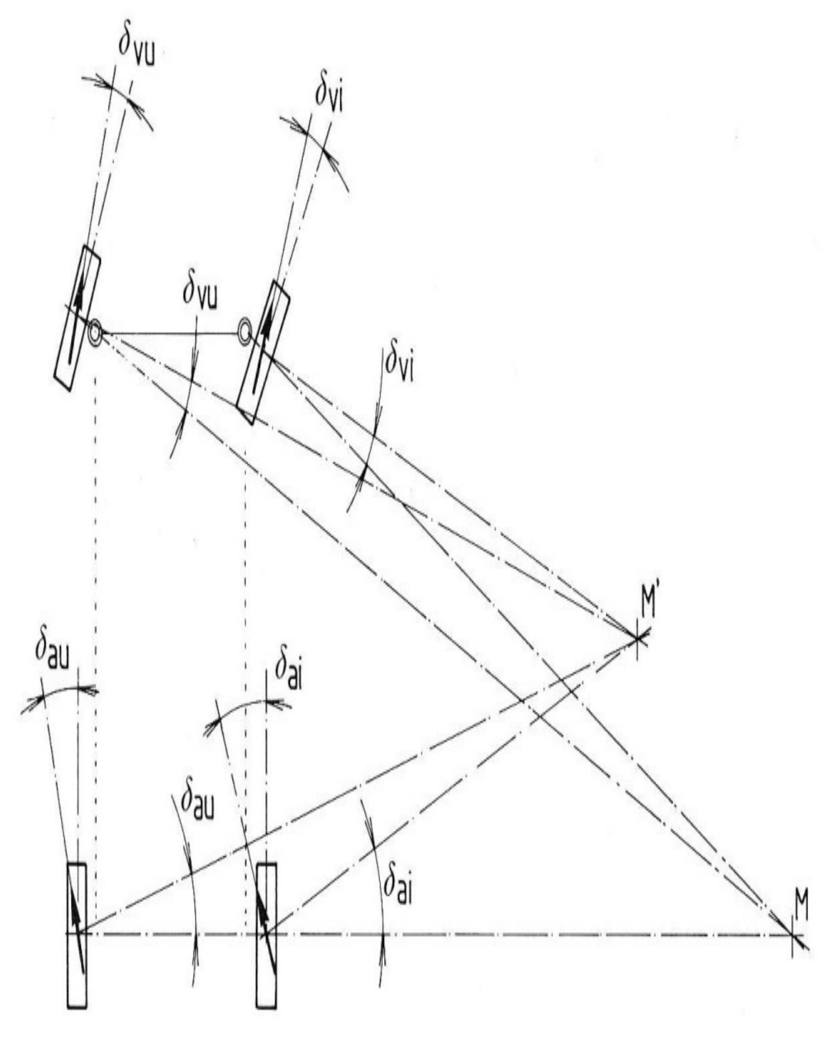
$$\cot\delta_R - \cot\delta_L = \frac{T}{L}$$

Ackerman condition for front-wheel-steering vehicles

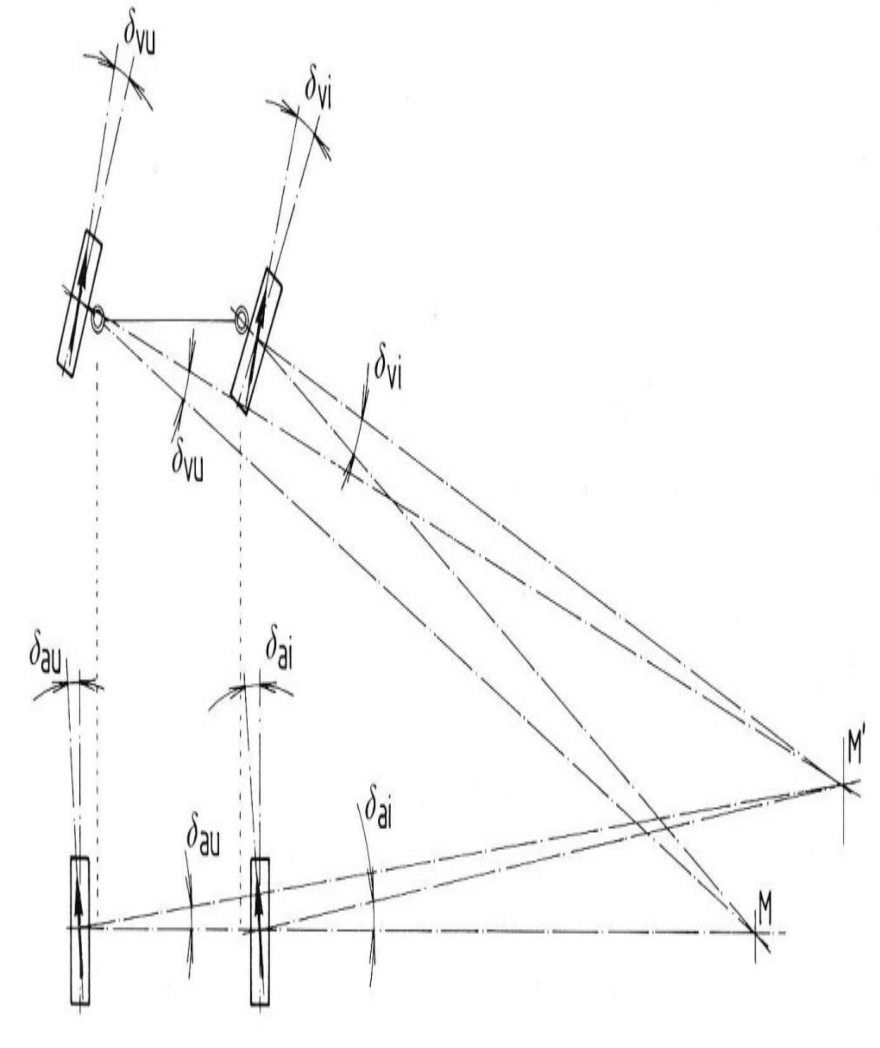


Steering Dynamics

Oversteer



Understeer



For the case of steady-state circular motion, the conservation of angular momentum

$$F_{\mathrm{sf}} \cdot l = m \cdot a_{\mathrm{y}} \cdot l_{\mathrm{r}} \qquad F_{\mathrm{sf}} = c_{\mathrm{sf}} \cdot \alpha_{\mathrm{f}}$$

$$F_{\mathrm{sr}} \cdot l = m \cdot a_{\mathrm{y}} \cdot l_{\mathrm{f}} \qquad F_{\mathrm{sr}} = c_{\mathrm{sr}} \cdot \alpha_{\mathrm{r}}$$

$$c_{\mathrm{sf}} \cdot \alpha_{\mathrm{f}} = m \cdot a_{\mathrm{y}} \cdot \frac{l_{\mathrm{r}}}{l} \qquad \alpha_{\mathrm{f}} = \delta + \beta - \frac{l_{\mathrm{f}} \cdot \dot{\psi}}{v}$$

$$c_{\mathrm{sr}} \cdot \alpha_{\mathrm{r}} = m \cdot a_{\mathrm{y}} \cdot \frac{l_{\mathrm{f}}}{l} \qquad \alpha_{\mathrm{r}} = \beta + \frac{l_{\mathrm{r}} \cdot \dot{\psi}}{v}$$

$$c_{\mathrm{sf}} \cdot \left(\delta + \beta - \frac{l_{\mathrm{f}} \cdot \psi \cdot}{v} \right) = m \cdot a_{\mathrm{y}} \cdot \frac{l_{\mathrm{r}}}{l}$$

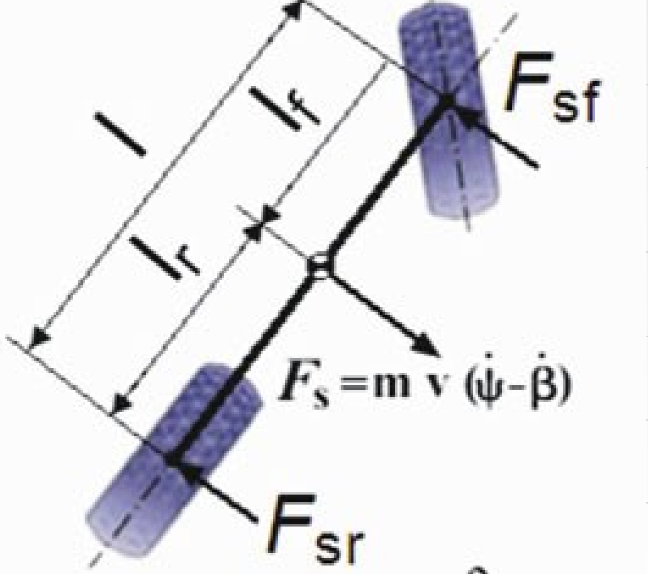
$$c_{\mathrm{sr}} \cdot \left(\beta + \frac{l_{\mathrm{r}} \cdot \dot{\psi}}{v} \right) = m \cdot a_{\mathrm{y}} \cdot \frac{l_{\mathrm{f}}}{l}$$

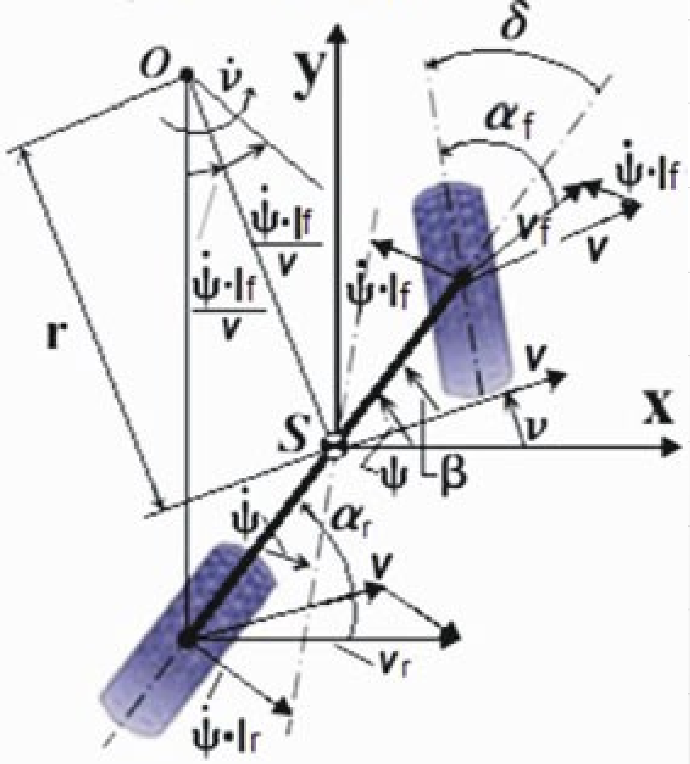
$$\delta = \frac{m}{l} \cdot a_{\mathrm{y}} \cdot \left(\frac{l_{\mathrm{r}}}{c_{\mathrm{sf}}} - \frac{l_{\mathrm{f}}}{c_{\mathrm{sr}}} \right) + \frac{\dot{\psi}}{v} \cdot (l_{\mathrm{f}} + l_{\mathrm{r}})$$

$$\delta = \frac{l}{r} + \frac{m}{l} \cdot \left(\frac{l_{\mathrm{r}}}{c_{\mathrm{sf}}} - \frac{l_{\mathrm{f}}}{c_{\mathrm{sr}}} \right) \cdot a_{\mathrm{y}}$$

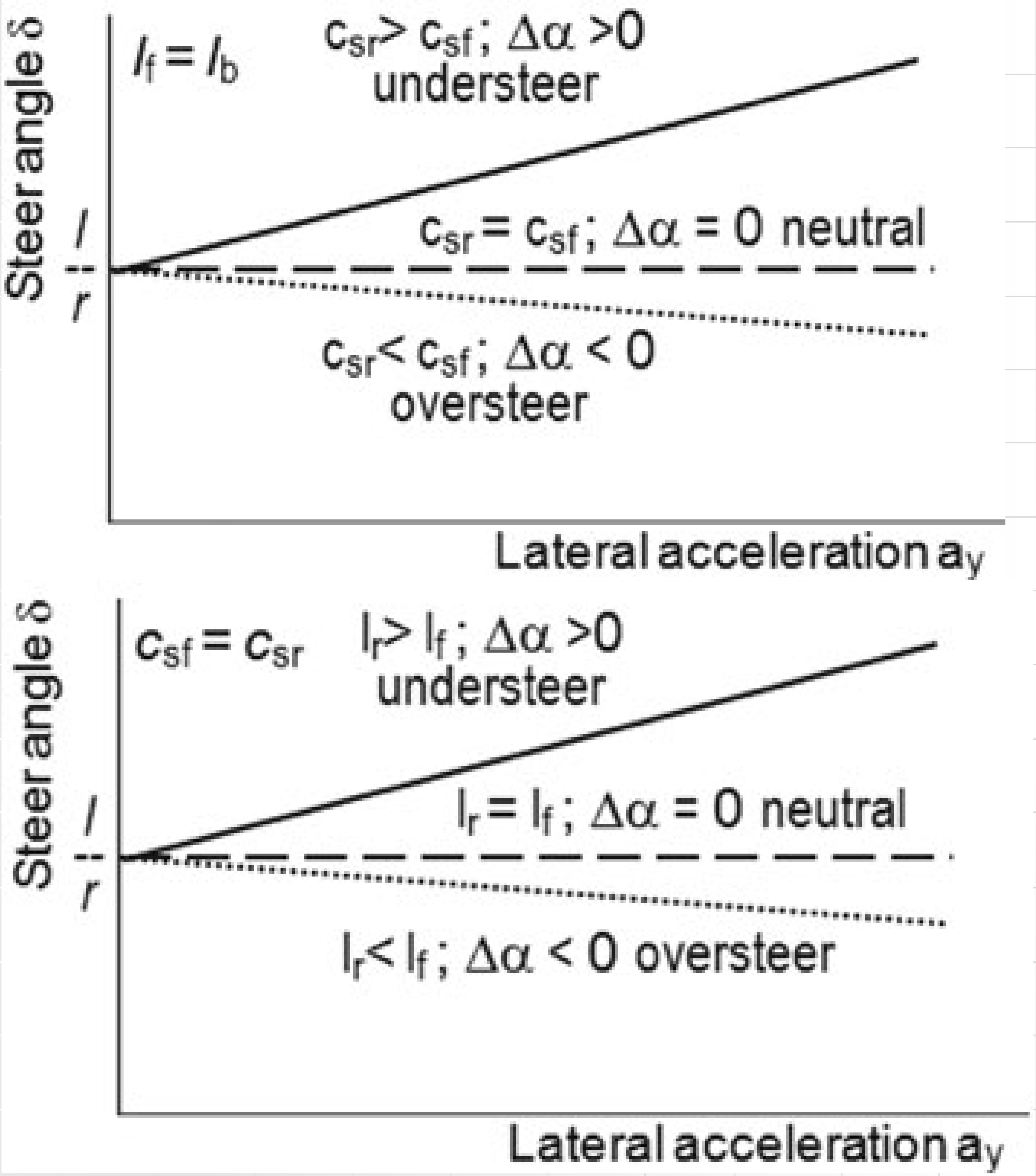
- v vehicle velocity,
- r radius of curvature (instantaneous),
- $\dot{\nu}$ angular velocity of the vehicle's center of gravity along the radius of curvature,
- $\dot{\psi}$ yaw rate (angular velocity of the vehicle about its z-axis),
- $\dot{\beta}$ slip angle velocity (change in angle between vehicle velocity vector at the center of gravity and vehicle longitudinal axis),
- α sideslip angle (angle between the tire's longitudinal axis and its velocity vector),
- δ steering angle at the front wheels (angle between the vehicle's longitudinal axis and the longitudinal axis of the tire).

- This equation shows that to keep the vehicle on course, the driver must apply a steer angle that not only satisfies the geometric conditions of the curve, but also compensates for the difference in sideslip angles Δα between the front and rear axles
- The difference in sideslip angles, which is dependent on both the tires and the vehicle itself, can also be referred to as a vehicle's self-steering behavior



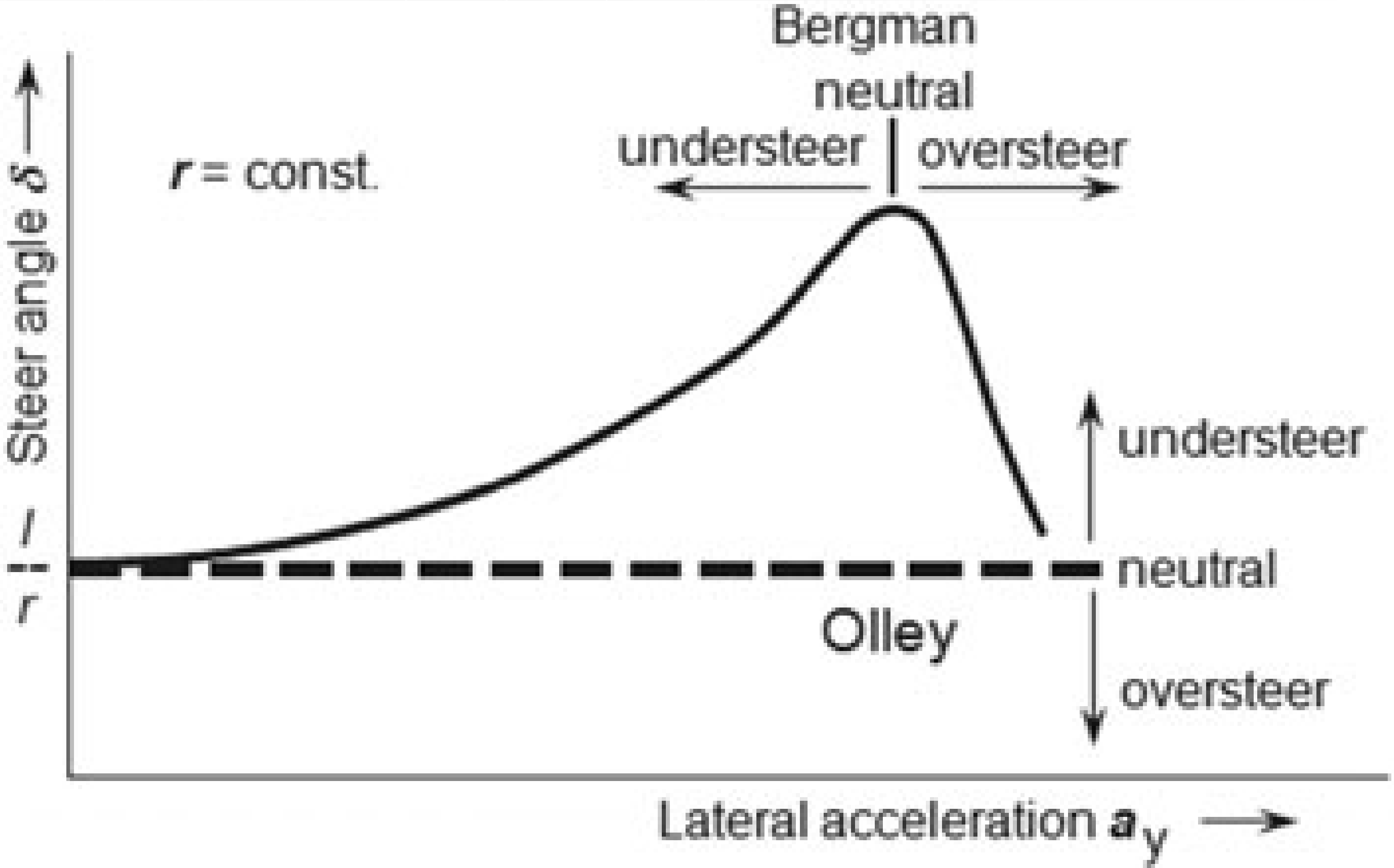


To negotiate a circular path according to Olley's definition, the driver must input a larger steering angle for a vehicle with understeer than for a vehicle with neutral handling characteristics



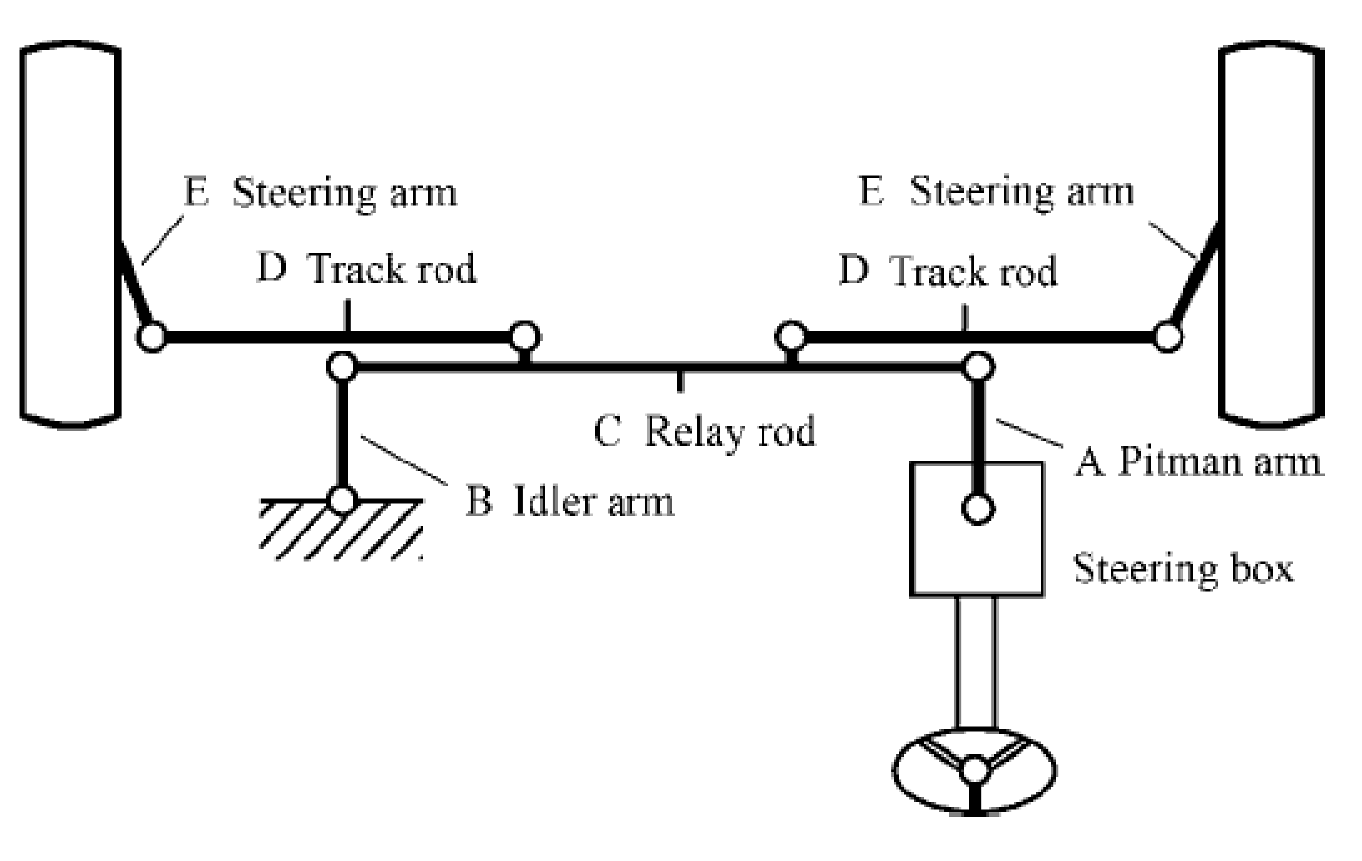
- A reduction in the required steer angle can occur rapidly in the region near the maximum steer angle
- Under this condition, the driver must reduce the steer angle Thus, the vehicle can said to be "oversteering"

$$\frac{\mathrm{d}\,\delta}{\mathrm{d}\,a_y} = \frac{m}{l} \cdot \left(\frac{l_r}{c_{sf}\left(\alpha_f\right)} - \frac{l_f}{c_{sr}\left(\alpha_r\right)} \right)$$



Steering mechanism;

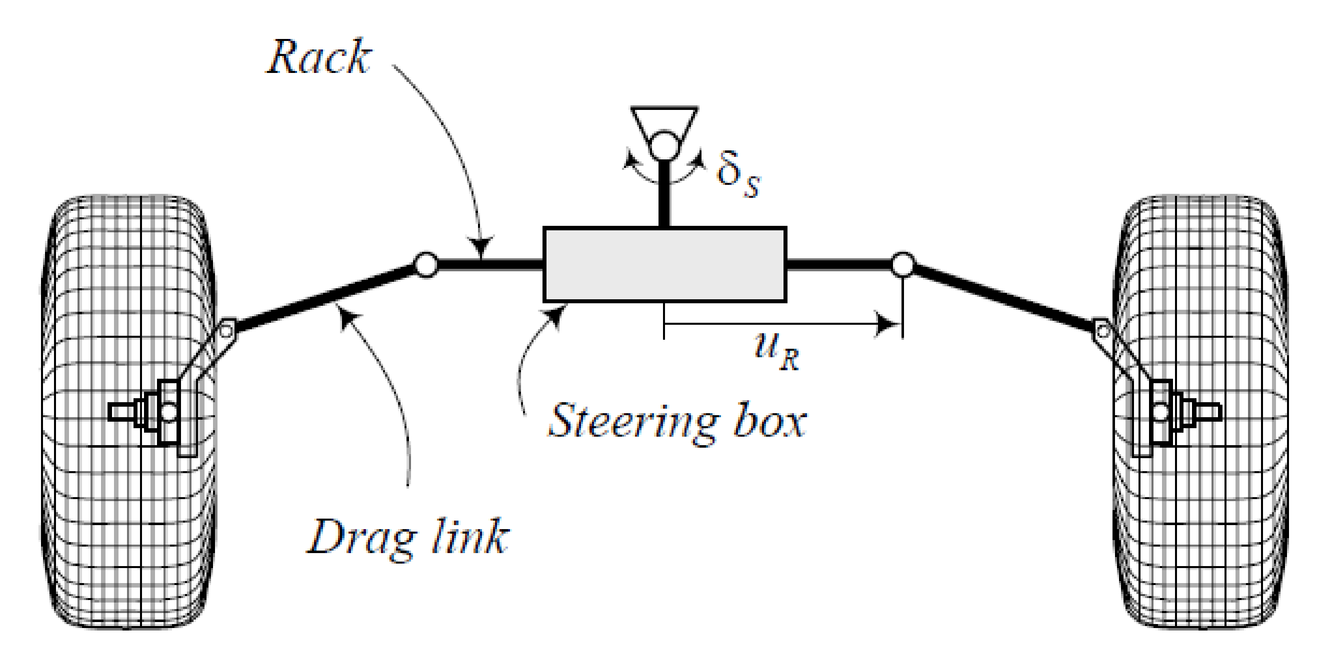
- steering system begins with the steering wheel or steering handle
- The driver's steering input is transmitted by a shaft through a gear reduction system, usually rack-and-pinion or recirculating ball bearings.
- The steering gear output goes to steerable wheels to generate motion through a steering mechanism.
- The lever, which transmits the steering force from the steering gear to the steering linkage, is called Pitman arm.
- The direction of each wheel is controlled by one steering arm.

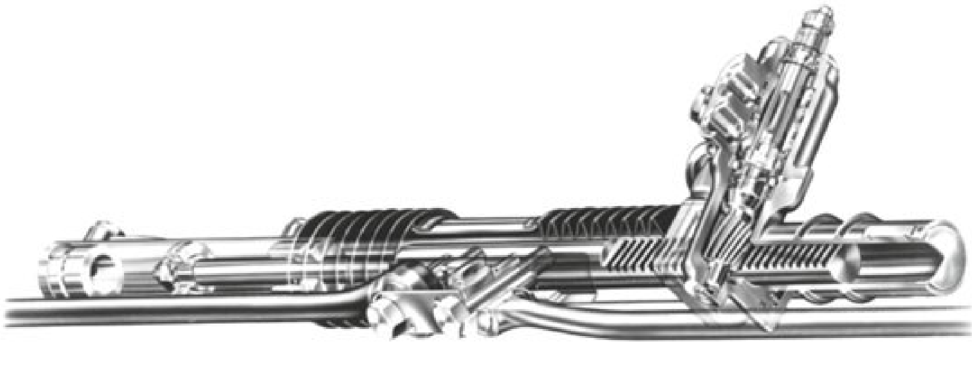


Steering mechanism;

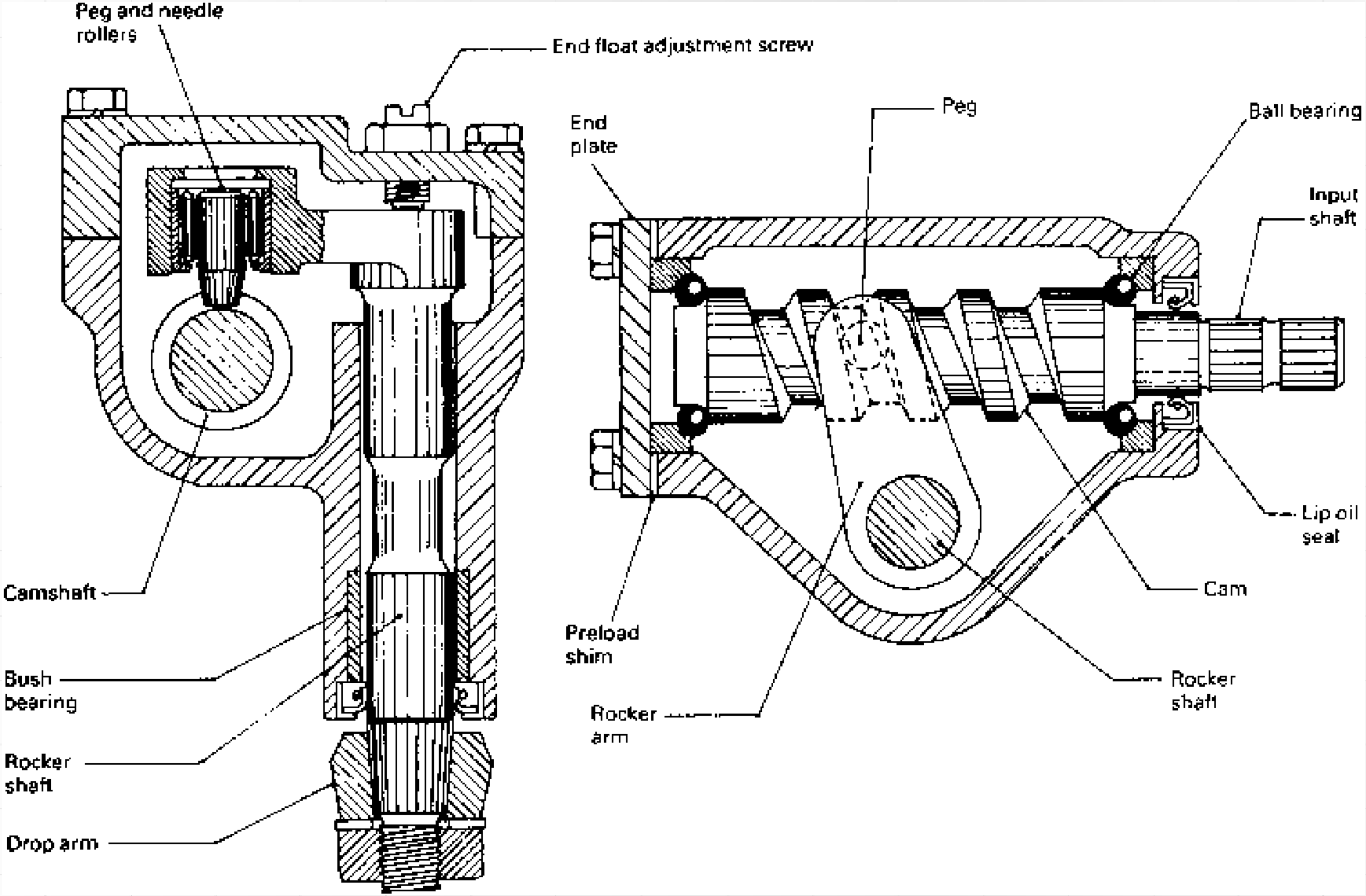
***Rack-and-pinion steering*.**

- The rack is either in front or behind the steering axle
- The driver's rotary steering command δ_S is transformed by a **steering box** to translation $u_R = u_R(\delta_S)$ of the racks, and then by the **drag links** to the wheel steering $\delta i = \delta i(u_R)$, $\delta o = \delta o(u_R)$. The drag link is also called the **tie rod.**
- The overall steering ratio depends on the ratio of the steering box and on the kinematics of the steering linkage





Cam and peg steering gearbox



Steering ratio

The overall steering ratio G is the rate of change of steering-wheel angle with respect to the average steer angle of the steered wheels, with negligible forces in the steering system, or assuming a perfectly rigid system, and with zero suspension roll

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- The steering ratio of Ackerman steering is different for inner and outer wheels.
- It has a nonlinear behavior and is a function of the wheel angle

$$G = \frac{\mathrm{d}\,\delta_{\mathrm{S}}}{\mathrm{d}\,\delta}$$

The mean overall steering ratio is

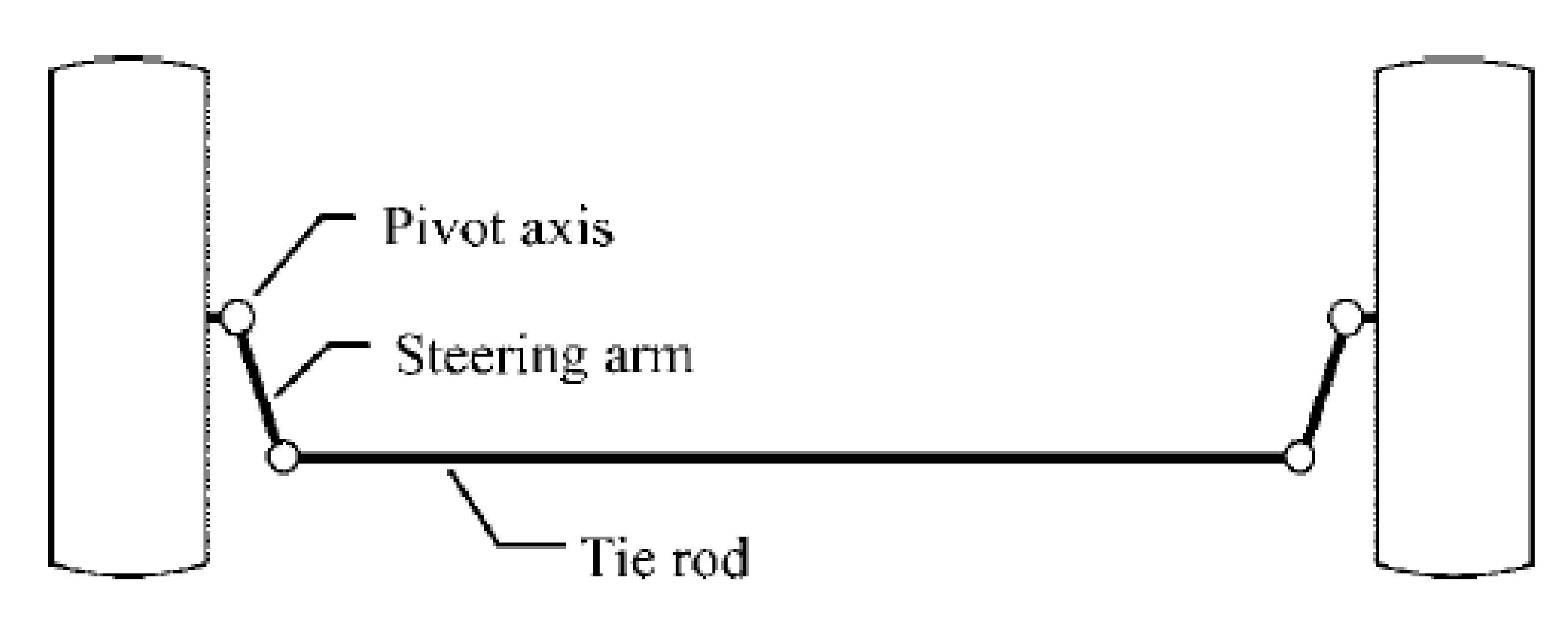
$$G_{\mathrm{m}} = \frac{\delta_{\mathrm{S}}}{\delta}$$

For a linear system G and G_m are equal and constant. In this case it is sometimes convenient to introduce the reference steer angle

$$\delta_{\mathrm{ref}} = \frac{\delta_{\mathrm{S}}}{G}$$

Steering mechanism;

- One convenient way to obtain complete or partial Ackermann steer angles is to angle the steering arms inwards
- the actual Ackermann factor varying in a complex way with the arm angle, rack length, rack offset forward or rearward of the arm ends, whether the rack is forward or rearward of the kingpins, and with the actual mean steer angle.
- Ackermann factor close to 1.0 may require the projected steering arm intersection point to be at about 60% of the distance to the rear axle.



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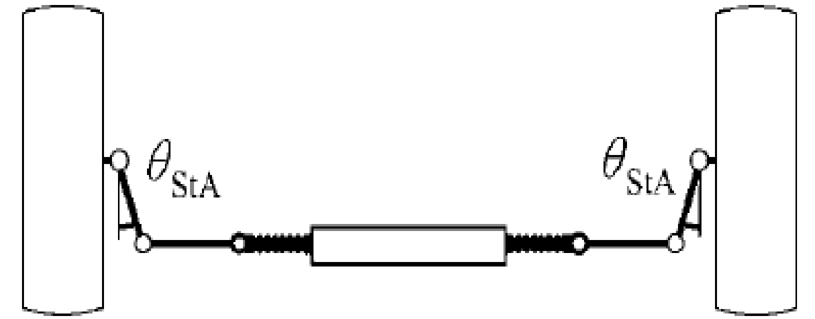
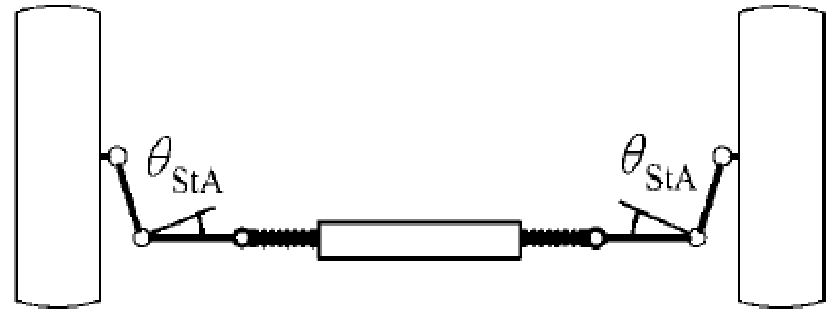
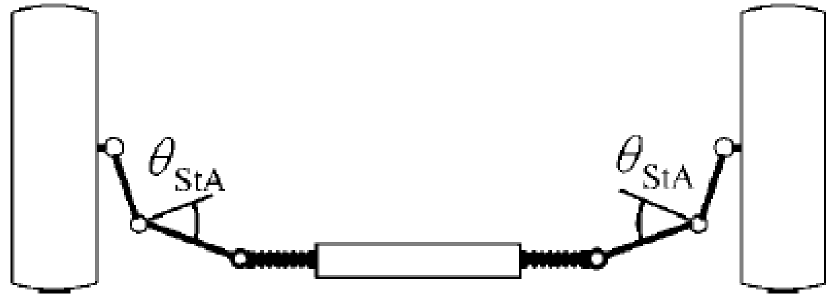
The steering arm angle usually considered important, as indeed it is with the aligned rack and track rods



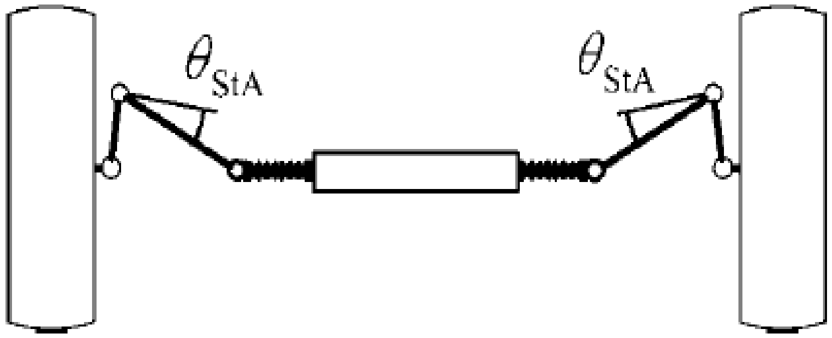
The angle that is actually important, the deviation from 90 at the steering arm to track rod joint



Angle opened up by moving the rack to the rear, increasing the Ackermann effect

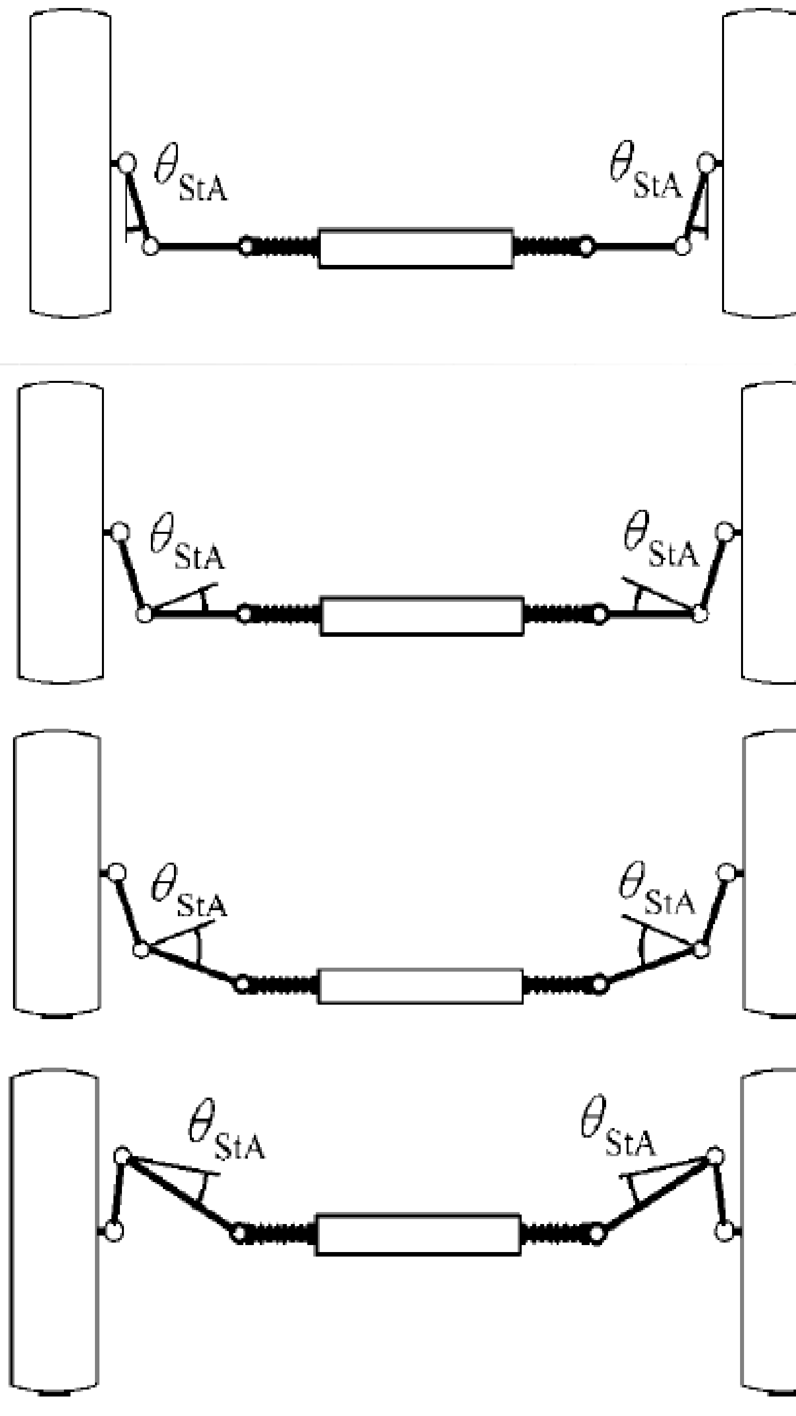


Inclination of the steering arms is not required at all, it can all be done by positioning the rack to give the necessary included angle. With this method, forward steering arms need not be angled out, and can even be angled inwards. although this increases the loads and wear on the rack



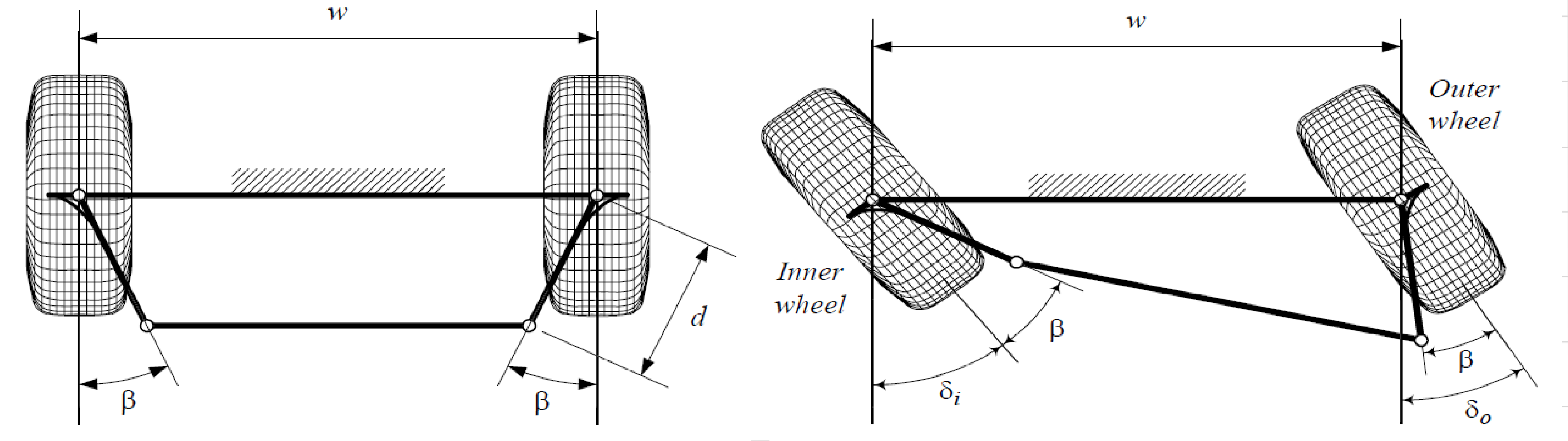
Considering the included angle between the track rod and the steering arm, with a conventional system, the steering linkage Ackermann effect angle, ideal (100%) Ackermann requires

$$\theta_{\mathrm{StA}} = \theta_{\mathrm{StA,I}} \approx 1.6\frac{T}{2L} = 0.8\frac{T}{L}$$

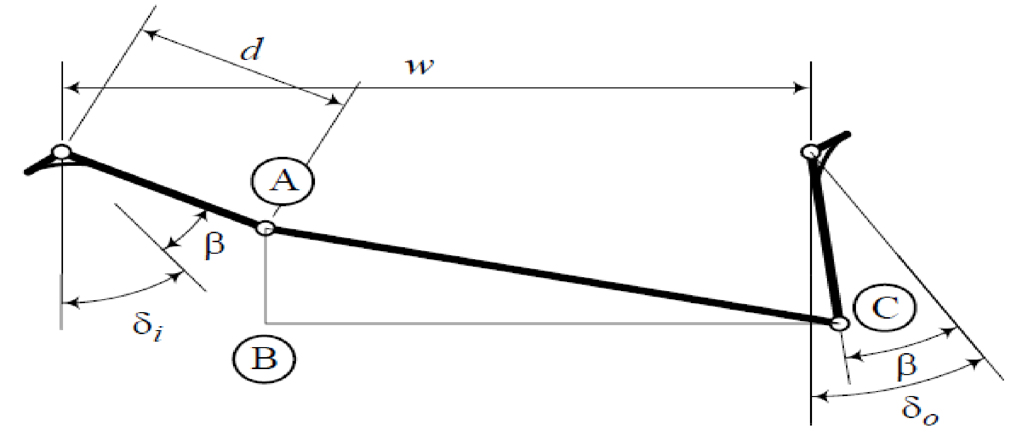


Steering mechanism;

trapezoidal



the triangle triangle ABC we can write



$$\begin{aligned}(w - 2d\sin\beta)^2 &= (w - d\sin(\beta+\delta_i) - d\sin(\beta-\delta_o))^2 \\ &\quad + (d\cos(\beta-\delta_o) - d\cos(\beta+\delta_i))^2\end{aligned}$$

hence

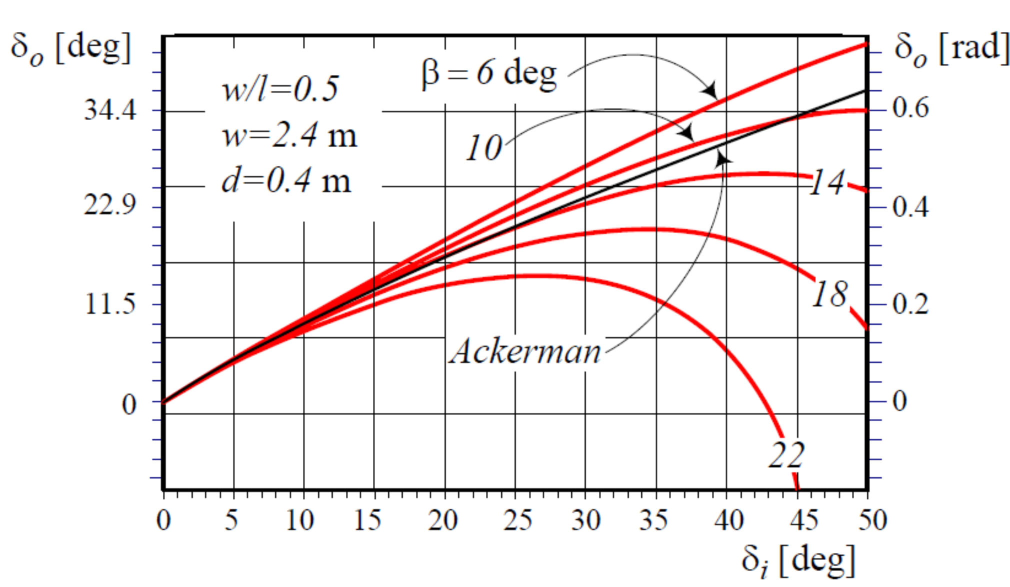
$$\begin{aligned}&\sin(\beta+\delta_i) + \sin(\beta-\delta_o) \\ &= \frac{w}{d} + \sqrt{\left(\frac{w}{d} - 2\sin\beta\right)^2 - (\cos(\beta-\delta_o) - \cos(\beta+\delta_i))^2}\end{aligned}$$

Steering mechanism;

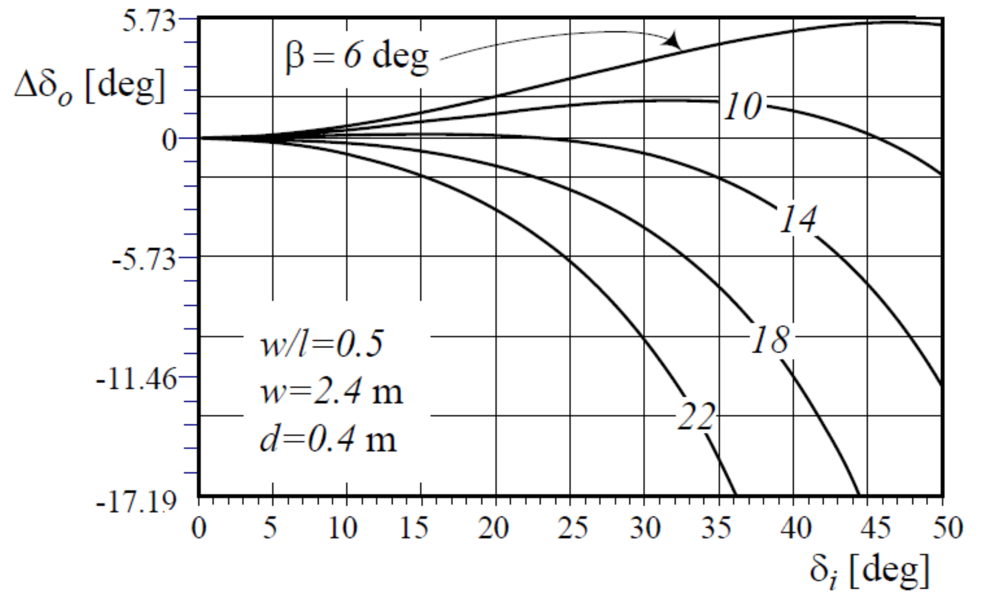
trapezoidal

- To examine the trapezoidal steering mechanism and compare it with the Ackerman condition, we define an error parameter e = $\delta_{Do} - \delta_{Ao}$.
- The error e is the difference between the outer steer angles calculated by the trapezoidal mechanism and the Ackerman condition at the same inner steer angle δ_i.

$$
\begin{aligned}
e &= \Delta\delta_o \\
&= \delta_{D_o} - \delta_{A_o}
\end{aligned}
$$

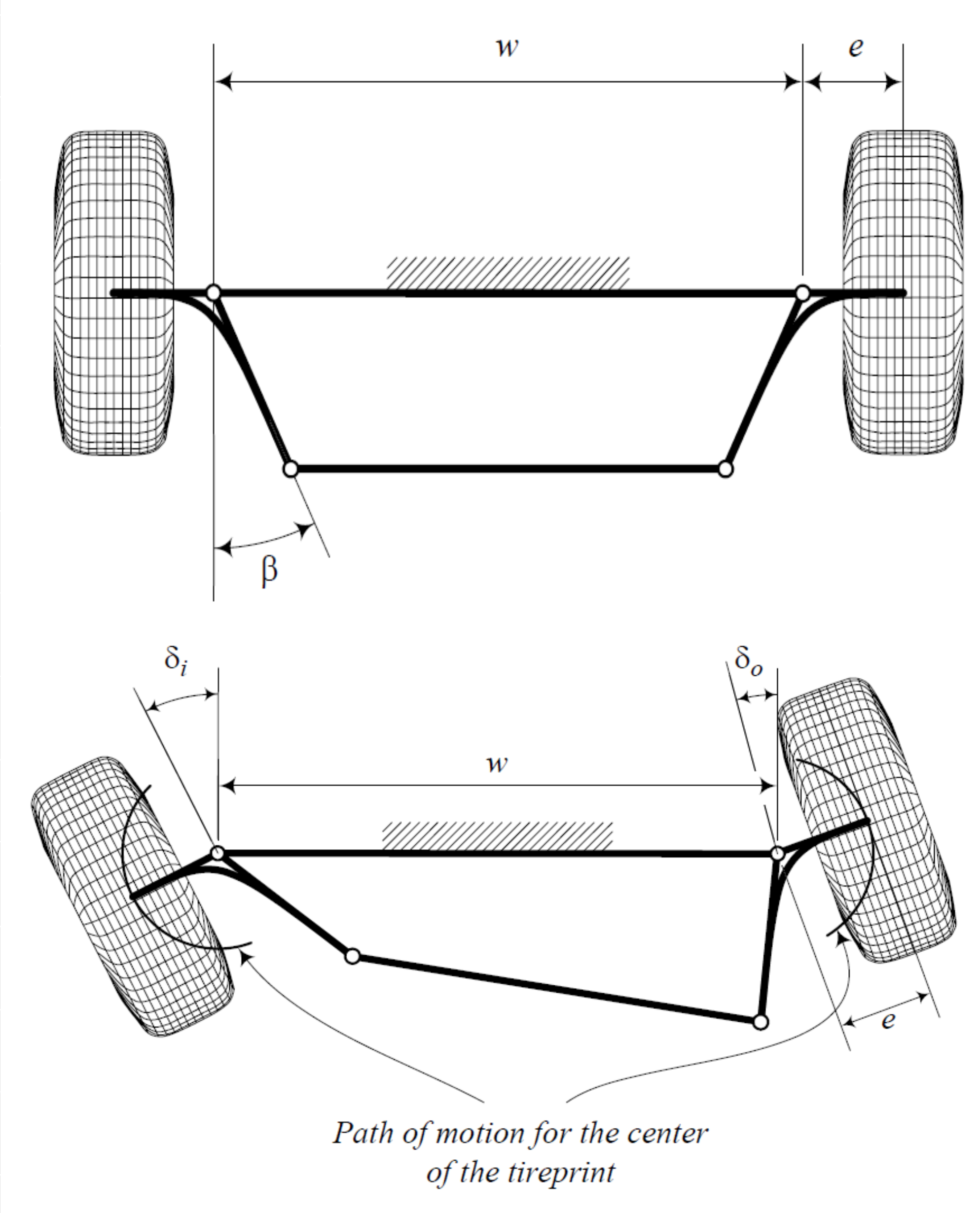


For a given l and w, a mechanism with β ≈ 10 deg is the best simulator of an Ackerman mechanism

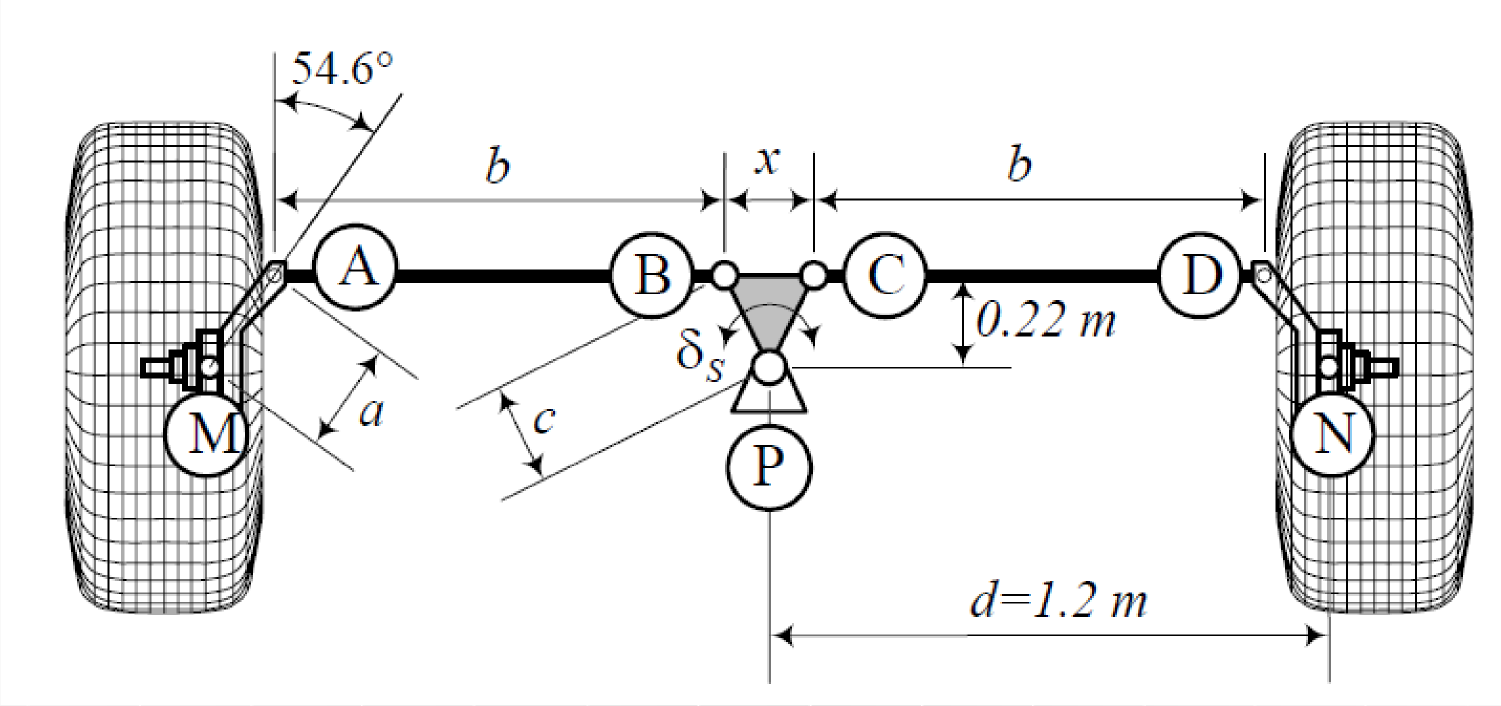


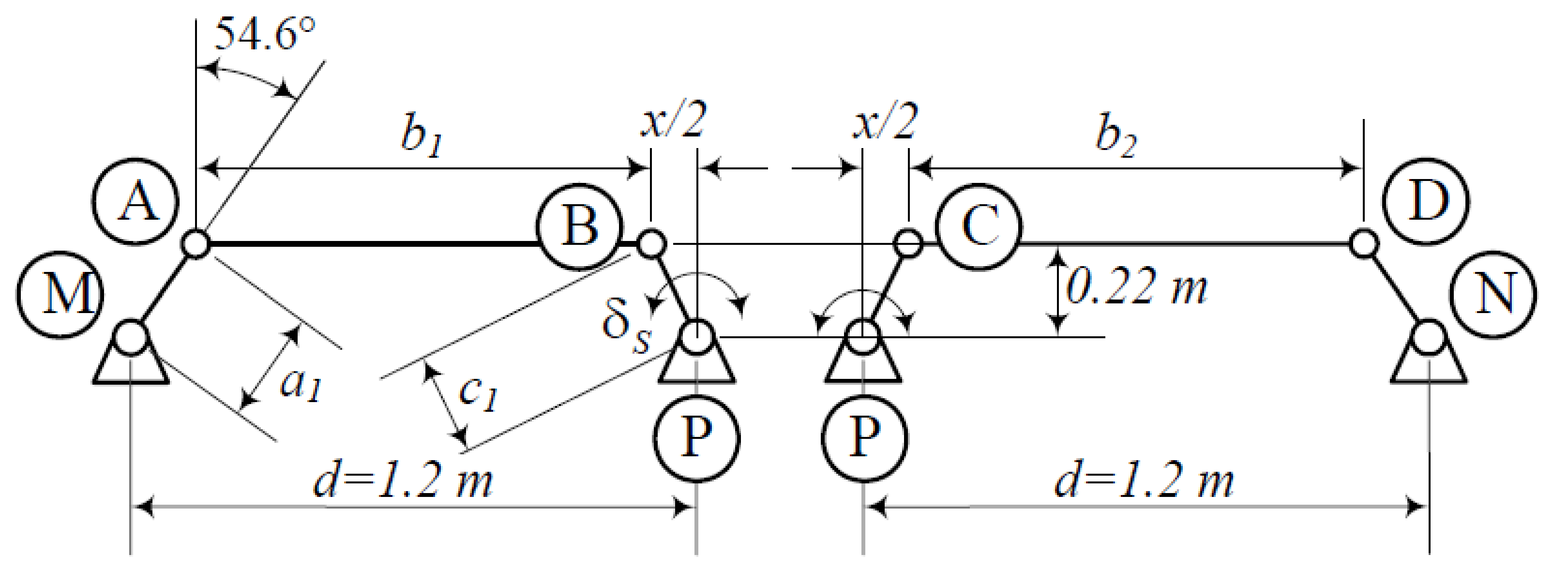
Offset steering axis

- path of motion for the center of the tireprint for an offset design is a circle with radius e equal to the value of the offset arm.
- not recommended for street vehicles, especially because of the huge steering torque in stationary vehicle.



Steering optimization

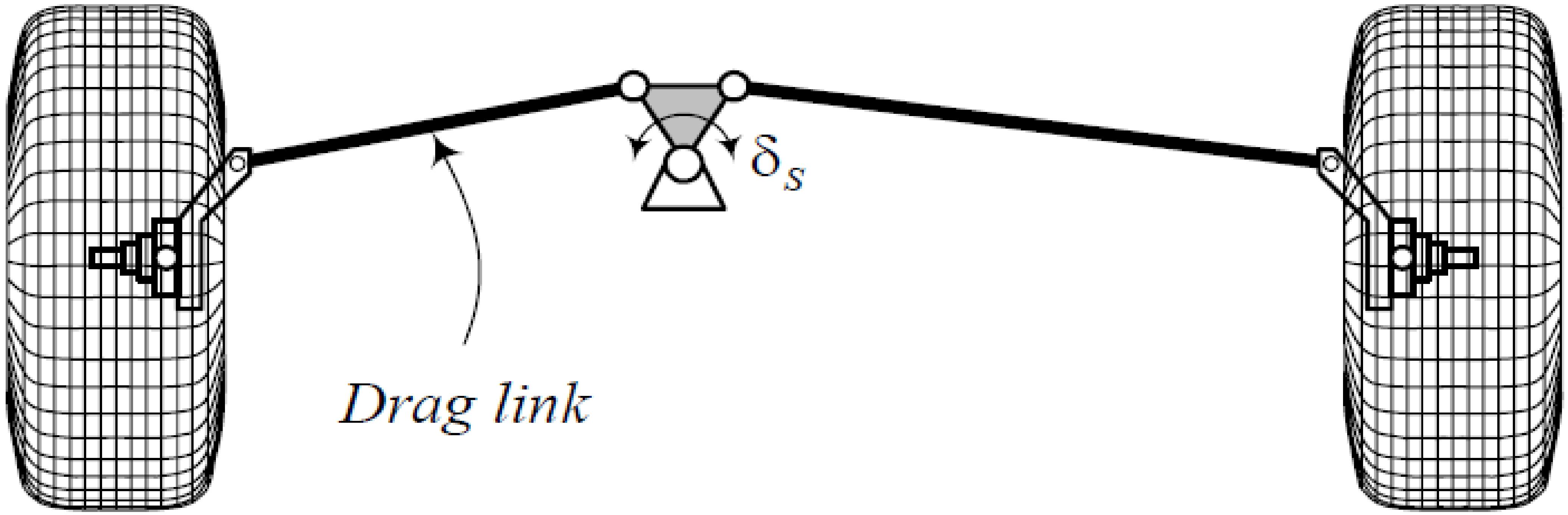




Steering Mechanisms

lever arm steering

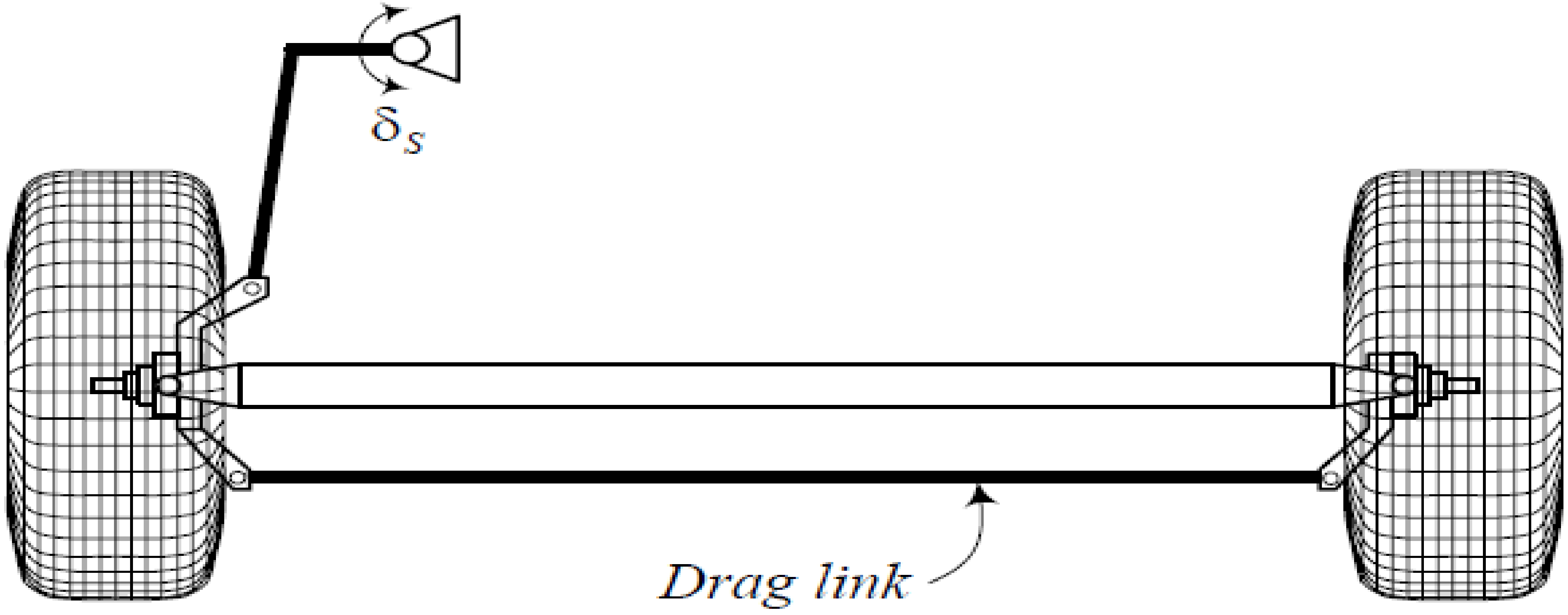
- Using a lever arm steering system, large steering angles at the wheels are possible
- This steering system is used on trucks with large wheel bases and independent wheel suspension at the front axle
- The steering box and triangle can also be placed outside of the axle’s center



Steering Mechanisms

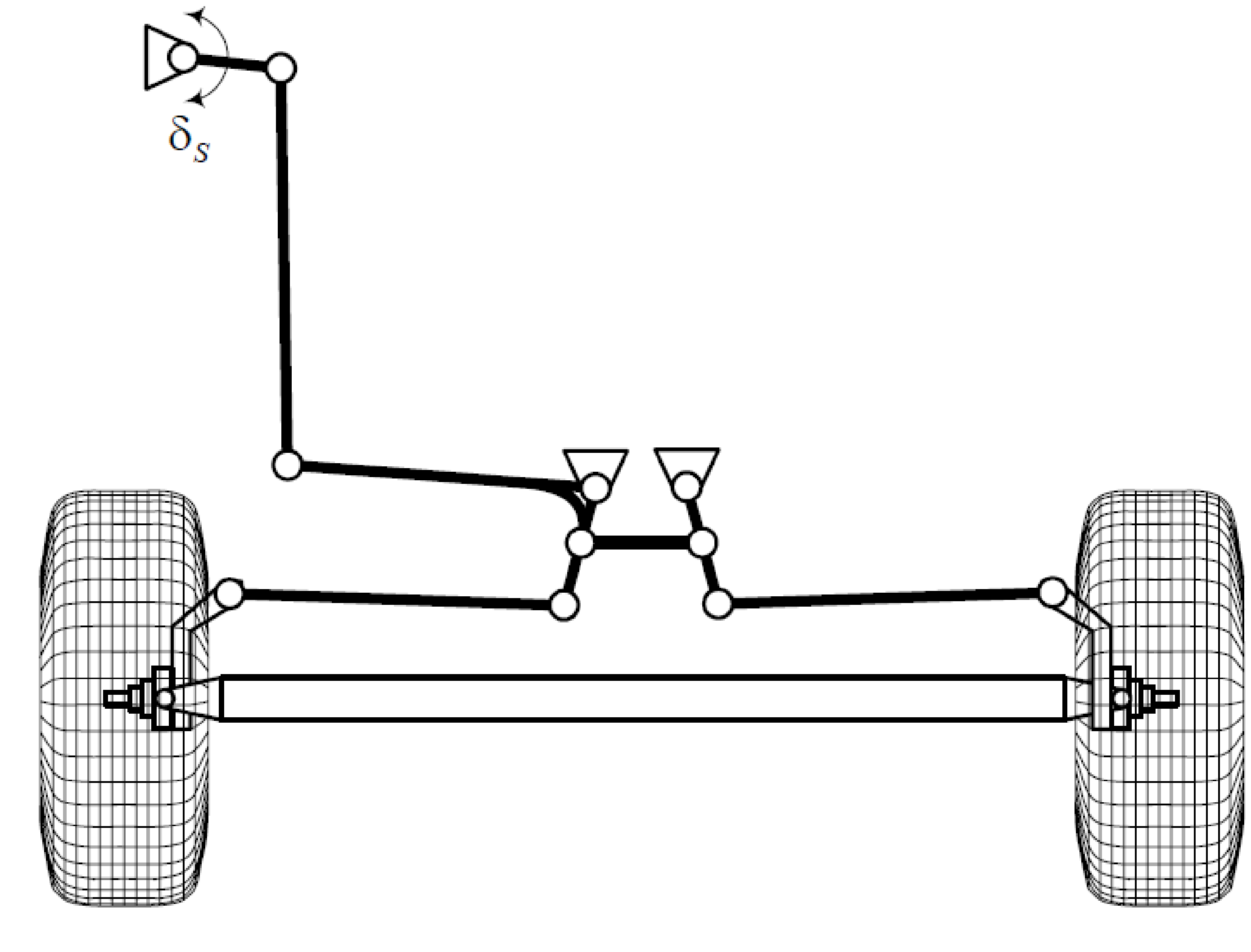
Drag link steering system

- Usually used for trucks and busses with a front solid axle.
- The rotations of the steering wheel are transformed by a steering box to the rotation of the steering arm and then to the rotation of the left wheel.
- A drag link transmits the rotation of the left wheel to the right wheel.



Steering Mechanisms
multi-link steering mechanism

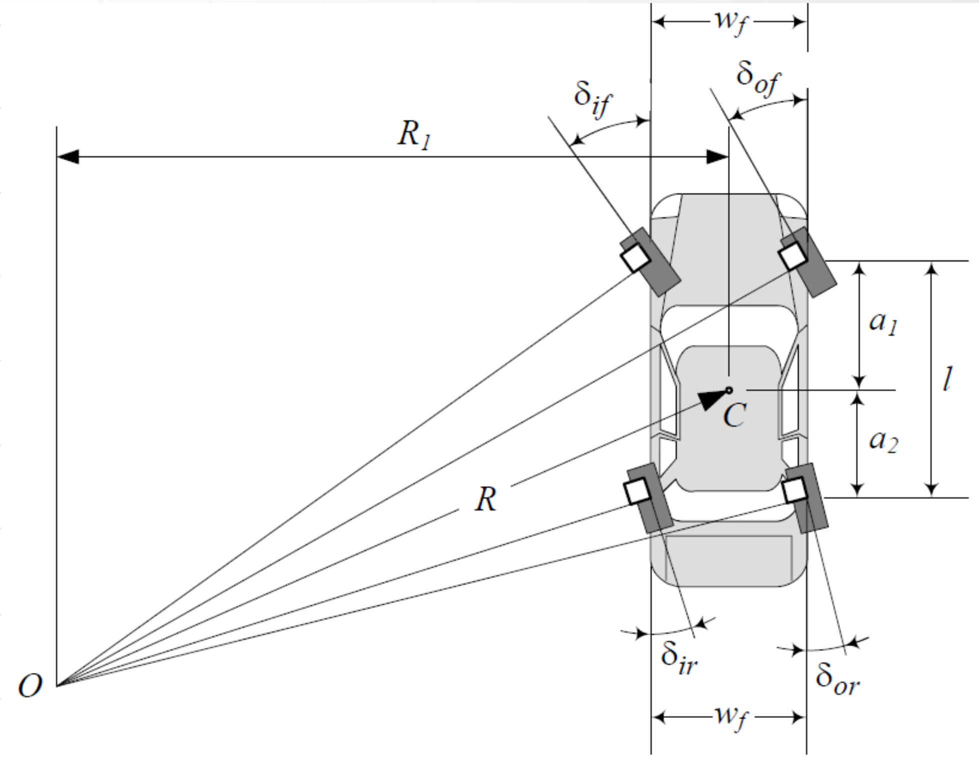
- The rotations of the steering wheel are transformed by the steering box to a steering lever arm.
- The lever arm is connected to a distributing linkage, which turns the left and right wheels by a long tire rod.



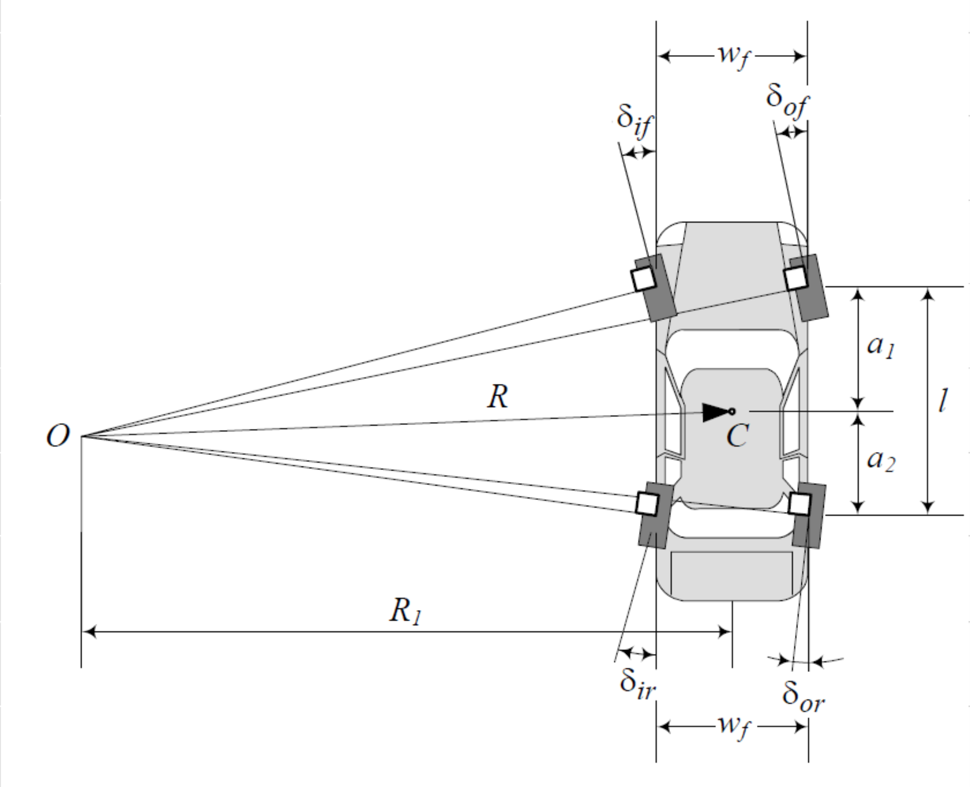
Steering Mechanisms

Four wheel steering

A positive four-wheel steering vehicle



A negative four-wheel steering vehicle.



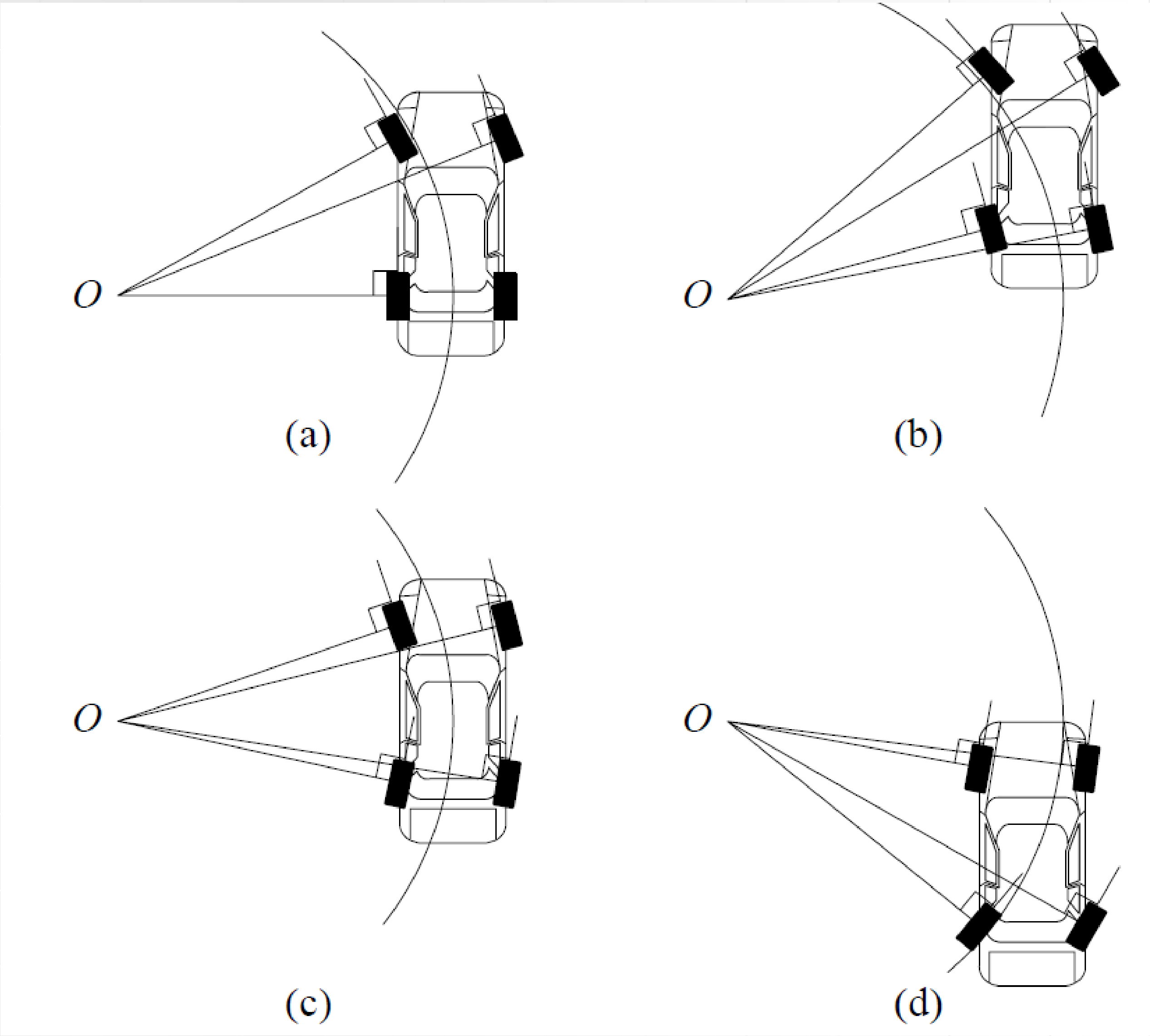
In a **positive** 4W S situation the front and rear wheels steer in the same direction, and in a **negative** 4W S situation the front and rear wheels steer opposite to each other

Steering Mechanisms

4WS vs FWS

- The turning center of a FWS car is always on the extension of the rear axel, and its steering length l_s is always equal to 1.
- The turning center of a 4W S car can be:
  1. ahead of the front axle, if $l_s < -1$
  2. for a F W S car, if $-1 < l_s < 1$ or
  3. behind the rear axle, if $1 < l_s$

$$
\begin{aligned}
l_s &= \frac{c_1 + c_2}{l} = \frac{l}{c_1} + 2c_s \\
&= \frac{1}{l}\left(\frac{w_f}{\cot\delta_{fr} - \cot\delta_{fl}} + \frac{w_r}{\cot\delta_{rr} - \cot\delta_{rl}}\right)
\end{aligned}
$$



Steering Mechanisms

Four wheel steering

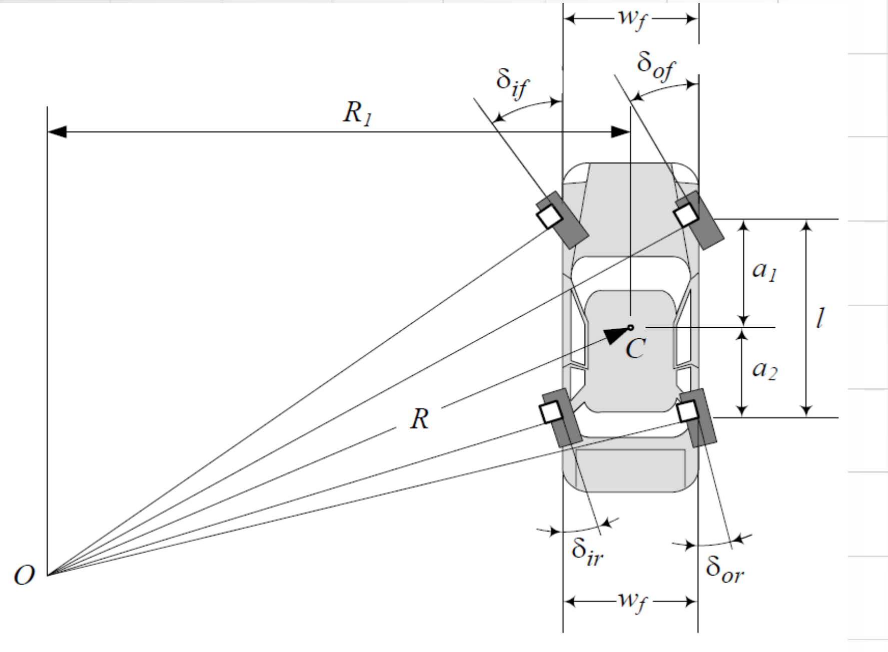
The kinematic condition between the steer angles of a 4W S vehicle is

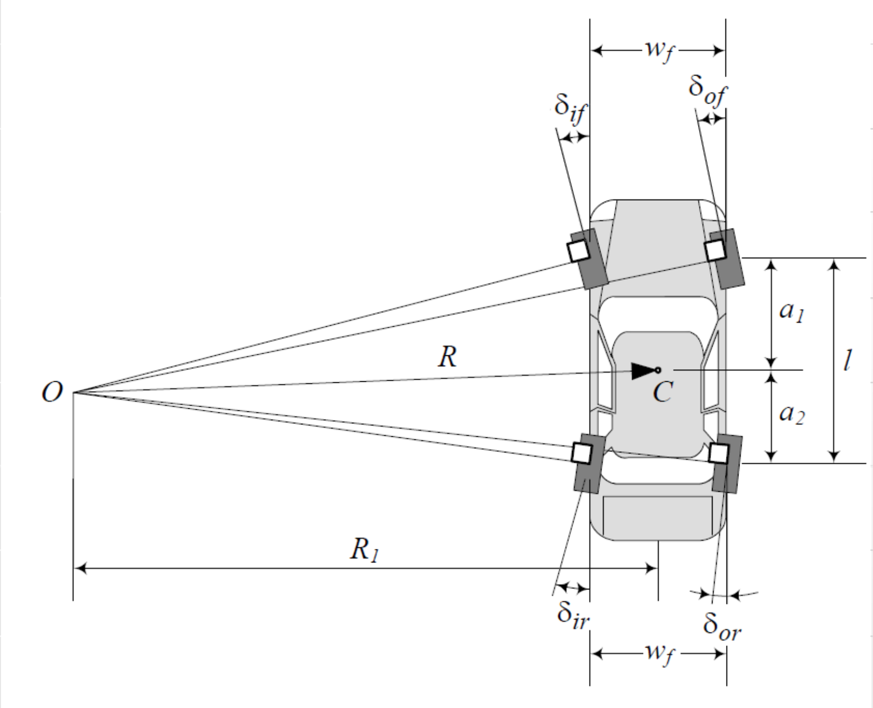
$$\cot\delta_{of} - \cot\delta_{if} = \frac{w_f}{l} - \frac{w_r}{l}\,\frac{\cot\delta_{of} - \cot\delta_{if}}{\cot\delta_{or} - \cot\delta_{ir}}$$

Or more general equation for the kinematic condition between the steer angles of a 4W S vehicle

$$\cot\delta_{fr} - \cot\delta_{fl} = \frac{w_f}{l} - \frac{w_r}{l}\,\frac{\cot\delta_{fr} - \cot\delta_{fl}}{\cot\delta_{rr} - \cot\delta_{rl}}$$

- The turning center is the curvature center of the path of motion.
- If the path of motion is known, then at any point of the road, the turning center can be found in the vehicle coordinate frame





Steering Mechanisms

Positive four wheel steering

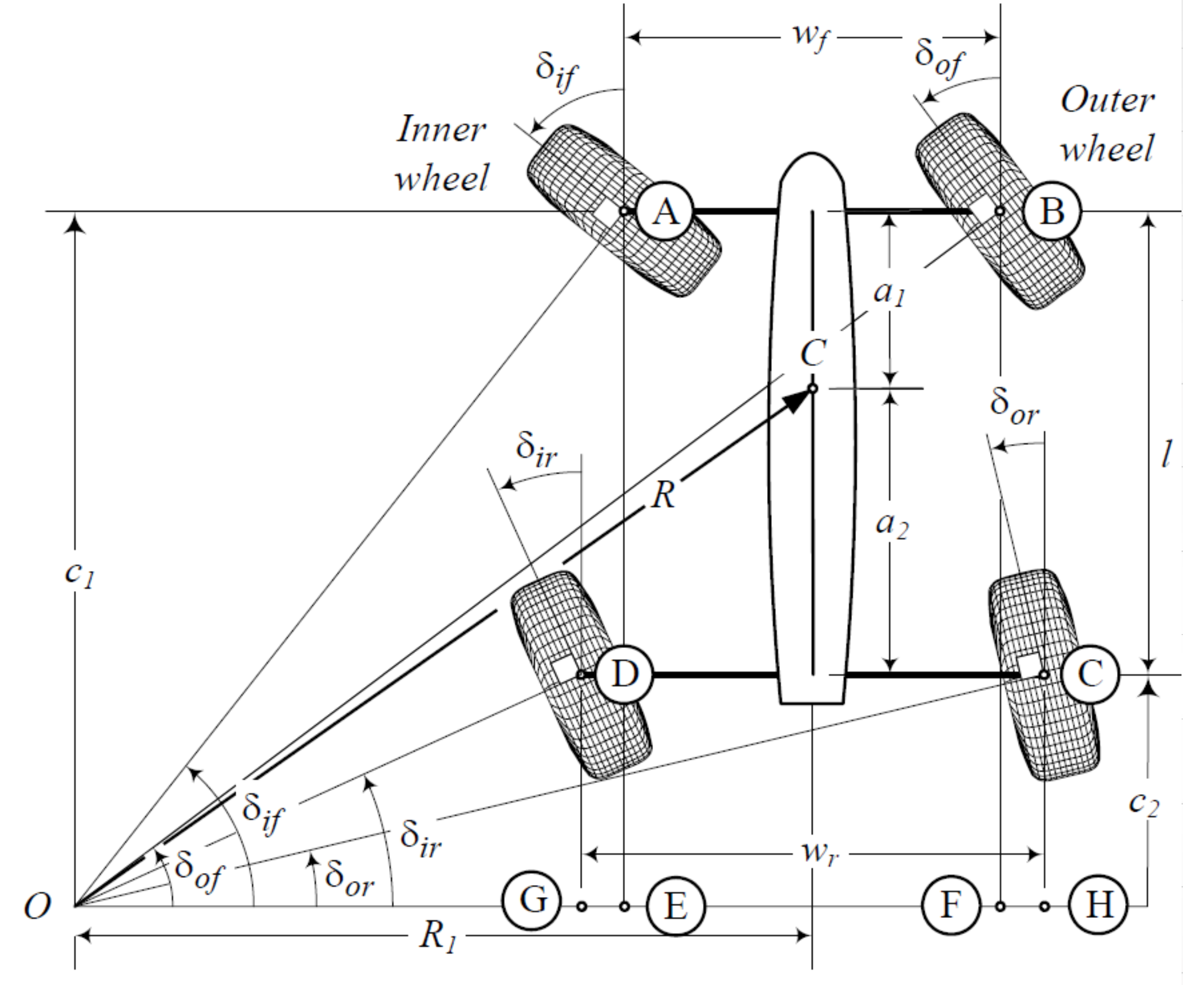
The front inner and outer steer angles δ_{if}, δ_{of} may be calculated from the triangles OAE and OBF, while the rear inner and outer steer angles δ_{ir}, δ_{or} may be calculated from the triangles ODG and OCH

$$\tan\delta_{if} = \frac{c_1}{R_1 - \frac{w_f}{2}}$$

$$\tan\delta_{of} = \frac{c_1}{R_1 + \frac{w_f}{2}}$$

$$\tan\delta_{ir} = \frac{c_2}{R_1 - \frac{w_r}{2}}$$

$$\tan\delta_{or} = \frac{c_2}{R_1 + \frac{w_r}{2}}$$



Elimination R_1

$$R_1 = \frac{1}{2}w_f + \frac{c_1}{\tan\delta_{if}}$$

$$= -\frac{1}{2}w_f + \frac{c_1}{\tan\delta_{of}}$$

$$\frac{w_f}{\cot\delta_{of} - \cot\delta_{if}} - \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}} = l$$

Steering Mechanisms

Negative four wheel steering

The front inner and outer steer angles δ_{if}, δ_{of} may be calculated from the triangles OAE and OBF, while the rear inner and outer steer angles δ_{ir}, δ_{or} may be calculated from the triangles ODG and OCH

$$\tan\delta_{if} = \frac{c_1}{R_1 - \frac{w_f}{2}}$$

$$\tan\delta_{of} = \frac{c_1}{R_1 + \frac{w_f}{2}}$$

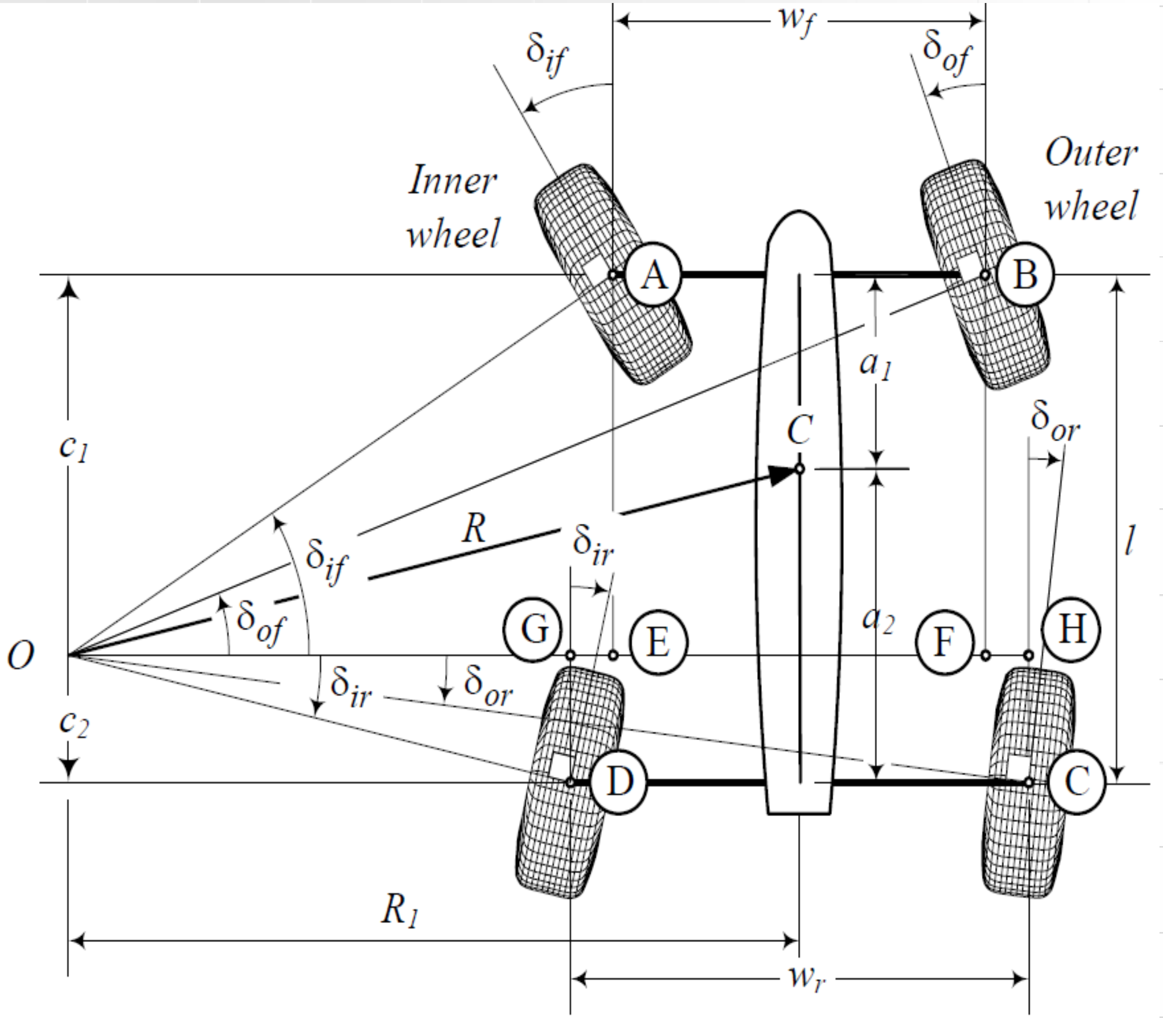
$$-\tan\delta_{ir} = \frac{-c_2}{R_1 - \frac{w_r}{2}}$$

$$-\tan\delta_{or} = \frac{-c_2}{R_1 + \frac{w_r}{2}}$$

Elimination R_1

$$R_1 = \frac{1}{2}w_r + \frac{c_2}{\tan\delta_{ir}} = -\frac{1}{2}w_r + \frac{c_2}{\tan\delta_{or}}$$

$$\frac{w_f}{\cot\delta_{of} - \cot\delta_{if}} - \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}} = l$$



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Steering Mechanisms

4W S factor; Steering length

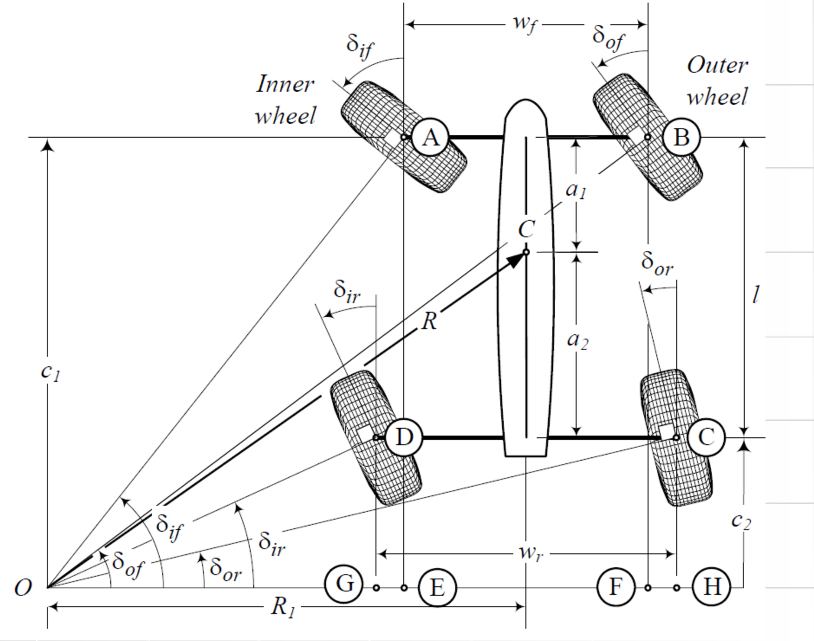
4W S factor **-** Longitudinal distance of the turning center of a vehicle from the front axle is c_1 and from the rear axle is c_2. We show the ratio of these distances by c_s and call it the 4W S factor

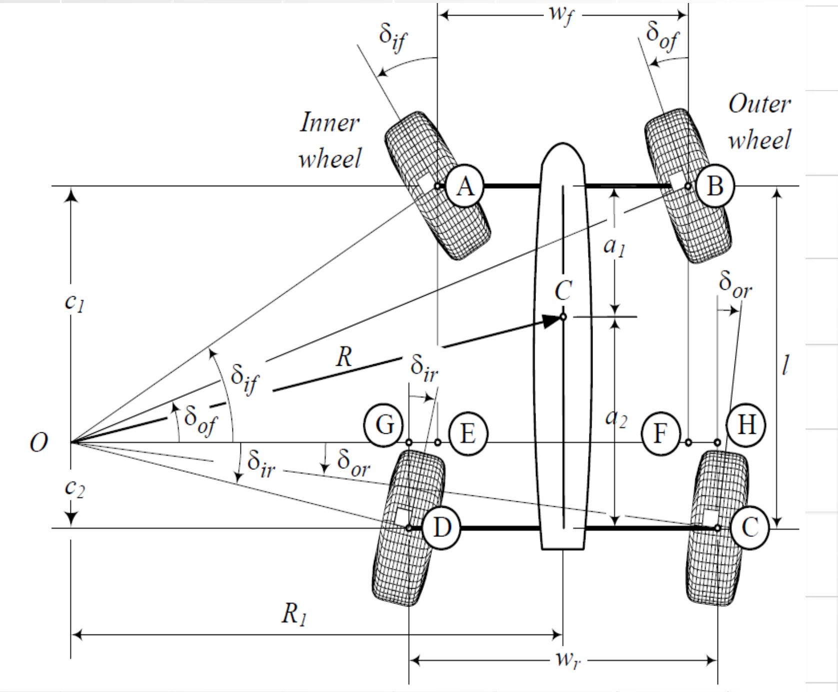
$$c_s = \frac{c_2}{c_1} \qquad = \frac{w_r}{w_f}\frac{\cot\delta_{fr} - \cot\delta_{fl}}{\cot\delta_{rr} - \cot\delta_{rl}}$$

Steering length For a 4W S vehicle, we may define a steering length l_s as

$$\begin{aligned} l_s &= \frac{c_1 + c_2}{l} = \frac{l}{c_1} + 2c_s \\ &= \frac{1}{l}\left(\frac{w_f}{\cot\delta_{fr} - \cot\delta_{fl}} + \frac{w_r}{\cot\delta_{rr} - \cot\delta_{rl}}\right) \end{aligned}$$

Steering length l_s is 1 for a F W S car, zero for a symmetric car, and −1 for a RW S car. When a car has a negative 4W S system then,





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Tendencies in steering system

Mass reduction;
Size reduction;
Materials replacements;
Angle optimization;
Electro-mechanical solutions;
4-wheel steering;
Force on steering wheel optimization;
Mechatronics in steering systems.

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